Restricted Choice

May 21, 2010 Homework Assignment (Due May 25th)

Today's lecture is at: http://www.lajollabridge.com/LJUnit/Education/Art_of_Being_Lucky.pdf

This homework assignment addresses an advanced topic called "restricted choice" which is often presented poorly and in a mathematically inaccurate fashion in the bridge literature. Here I'll try to do better.

Consider how to play the following suit, assuming *transportation* is available.

T 9 3 2 (dummy)

A K 6 5 4 (declarer)

The first step is to play the ace. If the defenders drop the 7 and 8, you will continue with the king, playing for the 2-2 break, the percentage play as demonstrated in the lecture. But suppose LHO drops the **jack** and RHO plays the 7. Now what? First off, LHO can never gain, even through deception, by playing the jack when holding J 8 or Q J 8. So you can be sure LHO started with either the singleton jack or the queen-jack doubleton. This means the initial full layout in the suit is one of these two cases.

	T 9 3 2			T 9 3 2	
Q J		8 7	J		Q 8 7
	A K 6 5 4			A K 6 5 4	

The winning play for the case on the left is to continue with the king, taking advantage of the 2-2 split. The winning play for the case on right is to get to dummy somehow (e.g. by playing to the ace in another suit, the *transportation*), and leading the ten (or nine), ducking if RHO plays small, and covering with your king if the queen appears. If you guess correctly, you will get all five tricks from your suit.

Q1: What is the *a priori* probability of each of the two layouts above, i.e. what is the probability that a four card suit splits 2-2 and has the layout on the left and what is the probability that a four card suit splits 1-3 and has the layout on the right? (Hint: these probabilities are not equal – see slide #8 from the talk).

Q2: Re-express the ratio of the *a priori* probabilities calculated in the previous question in term of odds, i.e. A:B where A and B are integers. (Hint: A and B are both under 20. Using them in place of the decimal percentages will simplify the calculations in the follow-up questions).

Your calculations may have convinced you that playing the king next is the percentage play. But wait a second – what should LHO play on the ace? The queen and jack are equivalent cards. There is no reason to always play the jack. Assume for the moment that when LHO holds the Q J that he will mentally flip a coin and play each card 50% of the time at the first trick.

Q3: What is the probability that leading the ten from dummy will pick up the entire suit under the assumption above?

Notice that on the left layout, LHO has a choice of cards to play at trick one. On the right layout, LHO has no choice. In the odd parlance of the bridge, his choice is "restricted".

In choosing to randomly play the jack or queen when holding Q J, LHO is pursuing what is termed a *mixed strategy* in the mathematical field of game theory, where John Nash, the subject of *A Beautiful Mind*, made early contributions directly related to mixed strategies. Many early pioneers in game theory, especially Johnny von Neumann, were interested in poker strategy where the correct choice is seldom to simply fold, call, or raise but rather to do all of these things a set percentage of the time, even if some of the percentages are small. Yes, you should even play a suited 76 once in a long while in Texas Hold'em; here game theory formalizes your belief that you must do this once in a long while to preserve your unpredictability.

Suppose LHO forgoes a mixed strategy in favor of lazily playing the lowest card each time. Eventually, his opponents will realize that he is always going to play the jack from Q J. This means that if LHO plays the queen it is sure to be singleton and declarer will be certain of being able to pick up the suit by going to dummy and leading the ten. But when he see the jack he will know that LHO is more likely to hold J x than a singleton jack and change strategy by playing for the drop.

Q4: What is the probability that declarer will pickup the entire suit against the lazy defender when all three restricted choice layouts are considered, i.e. LHO has jack singleton, queen singleton, or QJ. (Hint: don't forget to weight by the probability that each case occurs).

Q5: What additional percentage edge has the lazy defender given declarer by being lazy?

Q6: Does the lazy defender concede more than the house edge on roulette assuming: (a) European roulette where there is a zero; (b) American roulette where there is a zero and a double zero?

Note: in roulette, bets on black, red, and the 36 individual non-zero numbers would be even money bets except that the house collects all the money when the zero or double zero turns up.

Owing to symmetry, LHO does no better thinking he is being "tricky" by always dropping the Q from Q J. Once his opponents realize this, they will know when he plays the jack that it is guaranteed to be singleton. Now declarer will play the ten from dummy when LHO plays the jack at trick one and otherwise play for the drop. Only a mixed strategy maximizes the defender's chance.

Q7: Explain why the following suit combination is also a restricted choice situation:

K 9 3 (dummy)

A Q 8 7 6 (declarer)