Restricted Choice (with Answers)

May 21, 2010 Homework Assignment (Due May 25th)

Today's lecture is at: http://www.lajollabridge.com/LJUnit/Education/Art_of_Being_Lucky.pdf

	T 9 3 2			T 9 3 2	
Q J		8 7	J		Q 8 7
	A K 6 5 4			A K 6 5 4	

Q1: What is the *a priori* probability of each of the two layouts above, i.e. what is the probability that a four card suit splits 2-2 and has the layout on the left and what is the probability that a four card suit splits 1-3 and has the layout on the right? (Hint: these probabilities are not equal – see slide #8 from the talk).

A1: For the left diagram there are 6 (i.e. 4 choose 2) possible doubletons and we are interested in just one of them, a 1/6 x 40.7% = 6.78% chance where 40.7% is the 2-2 split probability for four outstanding cards as shown on slide #5 of the presentation. For the right diagram there are four possible singletons (or tripletons) and we are interested in just one of them, a 1/4 x 24.9% = 6.23% chance where 24.9% is the 1-3 split probability for a four outstanding cards.

Q2: Re-express the ratio of the *a priori* probabilities calculated in the previous question in term of odds, i.e. A:B where A and B are integers. (Hint: A and B are both under 20. Using them in place of the decimal percentages will simplify the calculations in the follow-up questions).

A2: 6.78% : 6.23% = 12 : 11

6.78 / 6.23 ≈ 1.09. If you simply try to convert this decimal to a fraction using a calculator that has such a function it may not give you 12/11 due to the round off error since neither 40.7% nor 24.9% is an exact number. But using the hint, it should not take long to discover the 12 : 11 is the correct ratio.

If you want to be precise you can instead calculate the number of layouts for each case. For the left diagram this is nchoosek(4,2) * nchoosek(22,11) / 6 = 705432. For the right diagram this is nchoosek(4,1) * nchoosek(22,12) / 4 = 646646. The greatest common divisor (gcd) is 58786, which is easily found via Euclid's algorithm even if you are using pencil and paper. So the ratio 705432 : 646646 reduces to exactly 12 : 11.

Note: computing nchoosek() by hand is often not as scary as it seems because many numbers in the numerator and denominator cancel, e.g. (22,11) = 22 * 21 * 20 * 19 * 18 * 17 * 16 * 15 * 14 * 13 * 12 / 11 * 10 * 9 * 8 * 7 * 6 * 5 * 4 * 3 * 2. After noting that 22 / 11, 20 / 10, 18 / 9, 16 / 8, 14 / 7, 12 / 6, are all 2 and furthermore that 21 / 3 is 7 and 15 / 5 = 3, we are down to 2^6 * 7 * 19 * 17 * 3 * 13 / 4 * 2 which after canceling powers of two further simplifies to 2^3 * 3 * 7 * 13 * 17 * 19.

If you work it out this way for the ratio of the number of layouts you'll end up with:

$2^3 * 3 * 7 * 13 * 17 * 19 : 2 * 7 * 11 * 13 * 17 * 19$

Canceling numbers common to either side yields 12 : 11. So who really needs a calculator? The backside of a napkin or two should suffice.

Your calculations may have convinced you that playing the king next is the percentage play. But wait a second…

Q3: What is the probability that leading the ten from dummy will pick up the entire suit under the assumption above?

A3: $11 / (½ * 12 + 11) = 11 / 17 = 64.7\%$.

Note that 64.7% is close to 66.6%, i.e. 2:1 odds. Many bridge books that discuss restricted choice state that the finesse has 2:1 odds in the basic restricted choice situation but as we see now that is not exactly correct.

Suppose LHO forgoes a mixed strategy in favor of lazily playing the lowest card each time….

Q4: What is the probability that declarer will pickup the entire suit against the lazy defender when all three restricted choice layouts are considered, i.e. LHO has jack singleton, queen singleton, or QJ. (Hint: don't forget to weight by the probability that each case occurs).

The occurrence ratio of the three cases above is 11 : 11 : 12. Against the lazy defender you win in two of the cases, thus $(11 + 12) / (11 + 11 + 12) = 67.6\%$

Q5: What additional percentage edge has the lazy defender given declarer by being lazy?

A5: $67.6 – 64.7 = 2.9\%$ (exact result is 1/34)

Q6: Does the lazy defender concede more than the house edge on roulette assuming: (a) European roulette where there is a zero; (b) American roulette where there is a zero and a double zero?

A6: Suppose the player bets $100 on red. In European roulette his expectation value is $200 * (18/37) = $97.30 since the payoff is 2:1 if the ball falls on a red number. Therefore the house edge is 2.7%, a little less than what the lazy defender concedes. In American roulette his expectation value is $200 * (18/38) = $94.74, a house edge of 5.26%

Q7: Explain why the following suit combination is also a restricted choice situation:

K 9 3   (dummy)

A Q 8 7 6 (declarer)

A7: Declarer is missing the jack and the ten. Declarer should start by playing the king. If both opponents play a small card, declarer should continue with ace and queen, playing for the 3-2 break. But if LHO drops the jack or ten at the first trick, it is more likely to be a singleton than from JT; therefore, declarer should run the 9 at the second trick, playing RHO for J 5 4 2 or T 5 4 2 as the case may be.