

## THE ULTIMATE COUNTING METHOD

The total efficacy of a Blackjack count depends on its correlation with (1) the true advantage or disadvantage of the remaining cards in the deck(s) for betting purposes and (2) the playing efficiency that it provides. Unfortunately, a count that does (1) well does not do (2) as well, and vice versa..

I had started out with the simple HI-LO count (23456+1, TJQKA-1) but wanted to look for something better.

Counts are described by how many “levels” they comprise. A count that gives all cards either a 0, 1, or -1 value is a “level 1” count, while one that gives 0, 1, -1, 2, or - 2 values to cards is a “level 2” count, and so on. The higher the level, the greater the betting accuracy but the more difficult the counting effort becomes.

Examining the count analyses for both betting and play efficiency in *The Theory of Blackjack* by Peter Griffin I saw that Stanford Wong’s moderately difficult “Halves” count has a betting accuracy correlation of 0.99. In whole numbers Halves counts aces and ten-cards as -2; 9 as -1; 2 and 7 as +1; 3, 4, and 6 as +2, and 5 as +5. It is more convenient, however, to divide all these numbers by two when counting, hence the name “Halves.” HI-LO’s betting correlation is 0.89 according to Griffin, so Halves is a somewhat better system for betting.

Looking at the playing efficiency of Halves, which is 0.58 according to Wong, it isn’t quite as good as the HI-LO count (0.59 according to Griffin). It’s close enough, however, to think that using Halves for both betting and play would be the simplest course.

But then I looked at the playing efficiency for other counts and saw there were a number of them with efficiencies of 0.61 and higher, up to 0.67. Could I perhaps employ two counts, one for betting and one for play? No, my brain isn’t up to that. But wait! I saw that HI-OPT I (3, 4, 5, 6 +1 and TJQK -1), with a 0.615 playing efficiency, is actually a subset of Halves. Suppose I were to keep the HI-OPT I count in my head and the difference between the two counts on the fingers of my right hand. Then I could use HI-OPT I for play and add the two counts (to get Halves count) when considering a bet. That shouldn’t be hard, with a little practice (but it was, requiring a lot of practice).

So how do I keep the count on my right hand? Easy. Plus counts are on the nails to begin with, tween-finger is ½, and on to the first knuckle for +5. Negative counts are done the same, but on the inner side of the fingers. Incidentally, I keep negative mental counts in French because it’s much easier to think, for instance, “trois” instead of “minus three.”

Griffin has an equation, very approximate, for measuring the combination of betting and play efficiencies to arrive at the potential profit available when K units are bet in favorable situations and one unit is bet otherwise::

$$[8(K-1) \times BE + 5(K+1) \times PE] / 1000$$

where BE = betting efficiency and PE = playing efficiency. “The formula suggests,” he says, “that the two efficiencies are equally important for a 1 to 4 betting scale and that betting efficiency is rarely more than one-and-a-half times as important as playing efficiency.”

Assuming a 1 to 4 spread, with Halves for betting (0.99 BE) and HI-OPT I for play (0.615 PE), I get an overall number of 0.039, or 3.91% profit per hand played. Griffin says to add 20% more for standard Las Vegas strip rules and 10% less for standard Reno rules. Of course the “standards” have changed since he wrote that.

The result for HI-LO BE (0.89) and PE (0.592) is 3.85%, only 0.06% worse.

If I apply the formula for Halves alone, with a PE of 0.58, for a 1 to 4 spread I get 3.83% profit per hand.

So maybe it isn't worth the effort. But wait! The playing efficiency of each count is much different for some situations, HI-OPT I is better than Halves for the majority of decisions but Halves is much better for some. If I were to employ both counts for playing purposes, choosing the appropriate one for each decision, PE would go up a bit (how much I don't know).

Conclusion: Use HI-OPT I for play and Halves for betting. Maybe the gain isn't great over a single count, but there is satisfaction in knowing that this is probably the best overall count system. When contemplating the splitting/resplitting of 10s, a lot of money may end up on the table, and it is comforting to know that Halves is pretty accurate for making that decision.

Peter Griffin's book told me how to determine which count is superior for a playing decision. I ended up using Halves only for decisions for which it was plainly superior, which are: hitting or standing with 16 vs 10, doubling with 10 or 9, and splitting 2s, 7s, 9s, and 10s. There were other marginal cases favoring Halves, but this was enough to remember.



If you are a gambler who always plays for cash, perhaps you have envied the player who walks up to the table, says “Marker please,” and gets a bunch of chips by merely signing a piece of paper. You can easily do the same, if you want. And you should, if you play a lot at one casino during a gambling trip. Markers are IOUs, like checks, paid out of your line of credit at the casino. Playing on credit is not only more convenient than carrying money around, but will also get you some free hospitality.

You apply for credit at a casino in the same way as at a department store. Get an application from the cashier’s cage and fill it out, or get one by mail if you wish. The most important information requested concerns your checking accounts: which banks, what account numbers. No other assets interest them, so don’t bother to show brokerage, savings, or money market accounts. They want to know if you can write a bank check for the amount of credit that you want, and they want to verify your balance. You will be asked to sign a statement that allows them to do that. No doubt they will check your credit history also. The form will also ask for the amount of credit you want. Often the credit approval will be for half the amount requested, so if you want \$10,000 credit ask for \$20,000.

Having large amounts of money in a checking account is silly, of course, so you will probably have to transfer funds to your checking account temporarily. Borrow some from a friend or relative if you wish (but have the resources to pay them back if you have to use it!). Avoid using a new account for this purpose. The casino will ask how long the account has existed, and you don’t want them to suspect a sting operation. The balance should be as large as is convenient, at least twice as much as the credit desired, preferably more.

It will take a week or two to get the credit approved. You should get a notice of approval in the mail or e-mail, but you may have to call the cashier to find your status. After the credit is approved, you can put the money back where it came from. They won’t check again unless you ask for a larger credit line or some of your checks have been returned. A warning: On future trips, don’t wait until you get back to hurriedly put money in your account to cover checks you have written. This may result in a notice of “Uncollected Funds” on checks they return to the casino. Uncollected funds represent bank deposits that are there but not there long enough (5 days?) to be paid out. Even though the checks will be honored when the casino sends them in again, they don’t like it, and it will be a black mark on your record.

So now you're ready to write markers. First, get acquainted with the casino host right away. Tell him/her you plan to give the hotel/casino your business, and ask for their business card. Several weeks before subsequent trips, write a note to the host saying you're coming and please reserve a room for you. Writing is better than calling, because a call will result in the host getting paged. Maybe you'll interrupt a meal or some other activity. There are other casino people who can grant you hospitality, but they are usually not so liberal as casino host.

After you check in to the hotel, go right to the casino cashier "cage" and tell them you will be using your credit line. Ask how much you are good for. You may find that a check written weeks ago has not yet cleared, reducing your credit line by that amount until it clears. If the casino uses computer terminals in the gambling pits, as nearly all do, your presence will be registered along with your current credit line. A manual system doesn't require this check-in with the cashier, but it doesn't hurt. It's embarrassing to ask for a marker at the table and be told to go check with the casino cashier.

Now you can write markers. When you are ready to play, go to your selected table and get your chips. If the pit boss is close by, just say, "May I have a marker please?" Otherwise ask the dealer, who will relay your request to someone in the pit. After getting your name, perhaps asking for identification, the pit boss will either consult the computer or call the cashier to check your credit line. Even if he knows you, he will probably want to check on your credit status since it shrinks with each marker you write.

When the pit boss is satisfied that you are okay, he will ask how much you want. The size of the marker should be at least 10% of your original line of credit; e.g., \$1000 for a \$10,000 line. Don't ask for odd amounts. The minimum marker should be \$500, and anything higher in \$1000 increments. The pit often has pre-printed markers for common amounts, and it annoys a pit boss to write a marker out for, say, \$700. You want to keep him happy, for reasons that will become clear.

After the marker is made out, you will be asked to sign it. Then the pit boss will tell the dealer to give you chips for the amount of the marker. Be careful! You are dealing with large amounts of money, and mistakes do happen. Watch intently as the dealer stacks the chips to be given you. You may be asked how you want it. The minimum should usually be \$25 chips. Asking for some \$5 chips is fine with a \$500 marker, but just get \$25 chips for \$1000. Similarly, it is okay to ask for some \$25 chips among the \$100 you will probably get for a \$5000 marker. The idea is to not look like a piker who is going to write a big marker and then play for small stakes. The dealer will presume that the smaller denominations are going to be used as "tokens" (tips), but tokening is not necessary unless you are eager to give the impression of being a big spender. The dealer can't help you (legally), why should you help the dealer? Remember that the tokens you give to a likable dealer are shared with the unlikable ones, since in nearly all casinos tokens go into a shared pool.

The pit boss will keep track of your action to some extent, particularly your average bet size. If he makes a mistake in this regard, you want it to be in your favor. Make your first few bets larger than average, to start off with a good impression. If the count goes plus, you can maintain or increase your bets. If it goes minus, you can cut back. The best time for cutting back is when the pit boss is busy at another table.

Your last few bets may also be important, because pit bosses know the ploy of starting with big bets, then reverting to small ones. Make sure he sees any large bets you put out. Call him over on some pretext if he hasn't noticed your big action. Your aim is to get him to over-estimate your average bet size. Why? Because that information, along with your time of play, is recorded, and determines how much hospitality you will get: free room, room plus coffee shop plus drinks, or full "RFB" (room/food/beverages) if your bets are quite large--all expenses paid, including expensive food and wine in the casino's gourmet restaurant. They might even refund your airfare, so have the receipt on hand. You and a guest may get invited to special party weekends, on the house.

You should play at least one-half hour per marker, preferably more, because playing time is recorded along with average bet size. Unless you bet \$100 or more at a crack, you will have to play 3-4 hours a day to get RFB. It is all right to change tables once or twice during a playing session, but don't change pits. They can't track your action if you jump around, and they worry about people writing markers and taking off without risking the chips. If you do change tables when the deck(s) go negative, let the pit boss know you are moving. He'll appreciate your courtesy.

There's no harm in looking like a loser, so most players will palm some chips during the play and transfer them to a pocket. The practice should not be overdone. It looks bad, and they don't like it. Remember that there is a video camera looking at every table and casino people walking around, just monitoring. Never ask for another marker with chips in your pocket--just quit, or use real money to finish up a hand. A good ploy is to have a companion openly ask for chips to play craps with when the pit boss isn't looking. Even if the "eye in the sky" sees it, the lack of secretiveness will make the exchange look innocent.

When you're through playing, ask the dealer to "color up" your chips to higher denominations. He will want to anyway, making it easier for the pit boss to see what you won or lost. The pit boss is supposed to record how much you walk away with, although your wins and losses are not recorded. He will also suggest that you buy back your marker if you have won. It is good form to do so, giving up an amount of chips equal to the marker, which is then returned to you. This in no way hurts your record of action. They like to see players using their entire credit line, not just a part. This is called "playing to the line." It is therefore better to have only as much credit as you plan to use on a trip.

Before checking out of the hotel, call the casino host and ask him to review your record for whatever hospitality he judges should be given you. Of course you will have charged all meals and drinks to your room, in case they get "comped." Remember that first impressions are important, so make the first trip a good one. Be polite, but don't be bashful. If your play warrants some freebies, you are entitled to them. If you get turned down for even a free room, just smile and ask for a discount on the room rate. They will always grant this request.

Frequent visitors get more hospitality than someone who comes once a year. Be sure to remind the host if he hasn't noticed how often you visit. Get credit at more than one place, so that they will feel impelled to compete for your business. Yes, credit at one place is known to the others.

You stand a better chance of getting RFB if your bill is moderate, so hold it down a little until you're sure of your standing. One way to do this is to ask the pit boss for freebies when your bet size is large. "Can you get us into the dinner show tonight?" will often get a positive response if he likes your play. He will make a call, then tell you it's all set. This probably won't show up on your bill, as pit bosses can give hospitality on their own account. You can do this for food too: "How about treating us to the brunch?" He will write you a chit to get you in free, and it won't reflect against your room charges. Don't feel like this is mooching; you have earned the hospitality by your play.

When cashing in chips after a winning session, the cashier may ask, "Do you have any markers to pay off?" It is probably better not to lie, even if you plan to borrow the money for a while. To avoid this situation, cash in your chips multiple times in smaller amounts, perhaps with the help of one or more companions. Some casinos have a neat little trick to keep track of what's going on: When a player cashes in \$100 (black) chips, the cashier picks up a phone to report the transaction. The call is not to a person, but to a recording! A video camera is witnessing the scene. Casino personnel can replay the cashier videotape and hear a voice summary of all large transactions. The purpose may be to monitor cashier honesty/accuracy, but a side benefit is seeing who is cashing in big money. Practical tip: Use the safety deposit boxes that most casinos make available to their guests free of charge. It's not wise to carry around large amounts of unneeded cash.

If you do bring large amounts of money home instead of buying back markers, watch out! Keep very good accounts of where you put every penny, and save all associated records--every marker, every receipt for payments, every deposit record. There is an organization called the IRS that may audit you someday. They will want to know where all those big deposits came from. Are you a drug dealer or something, with unreported income? As a corollary, remember to report and pay income tax on winnings that are in excess of losses. Gambling losses for one year cannot be carried over to the next, so forget that.

On your first visit you will be asked to write a check for all outstanding markers before you leave. After that you can ask them to bill you, if you wish. The statement will come in a week or two and you should pay it off promptly, within a month at the latest. After 45 days Nevada law says that the casino must forward any outstanding markers to your bank for payment--they are really counter checks, after all. They may do that before 45 days if you have agreed to pay sooner. After your payment is processed, they will mail your markers to your home or business (as indicated by you on your credit application). At that time, write a note to the casino host thanking them for their hospitality.

The next time you come to the hotel, you must have paid off any markers from the previous trip in order to play on credit. It doesn't matter to them that you used only 30% of your credit line two weeks ago, you can't play until that 30% is paid. Suppose you give them a check for that 30%. Until the check clears, your line of credit is 70% of the original. To avoid this situation, pay off markers quickly.

After several trips you will find that your freebies are automatic--you won't have to ask the host to take care of your bill. He will reserve your room with a special code that says, "No charge for this room," or perhaps, "No charge for this guest." The registration clerk will be glad to explain your status if you are in doubt. You will always have to pay tips that you may have put on your tab, and personal charges such as haircuts, massages, etc. The most efficient policy is to not charge such items to your room. Alternatively, you can provide a credit card on arrival to be used for that purpose. In either case you can check out quicker than when paying any charges with a check.

You can establish credit at more than one place, but there is a central credit agency that keeps all casinos informed of your accounts. If you use all your credit line at one place, you probably will be turned away at the next. That's easy to avoid--always leave a few thousand or so of unused credit when you go to another casino.

## *Funny Money*

By Marvin L. Master



The publication Las Vegas Advisor, authored by Tony Curtis, refers to the non-negotiable chips give out by some casinos as "funny money." The Nevada Palace, for instance will give you \$400 in funny money in exchange for \$198 in real money. You can't cash these chips in the casino cage, they must be lost at the tables. You play each chip one time only, not counting ties. If you lose a bet, the dealer takes the chip. If you win, he also takes the chip but gives you a "real money" chip of the same denomination. Winning 50% of your bets would give you 50% of the funny money face value in real money chips. Your Nevada Palace room would then cost you nothing, even after a \$2 tip for the dealer.

Should you expect to win 50% of your funny money bets? That depends. The Las Vegas Advisor estimates a 47.5% win rate at Blackjack. I don't know how they got that number, which represents a 5% casino advantage. The actual disadvantage is about 2.5%, caused by the even money payoff for a funny money blackjack instead of the standard 3 to 2. That represents a win rate of 48.75%. The \$400 funny money would earn an expected \$195 with flat betting and merely correct play ("basic strategy"). Card counters will do better, of course, but are there other ways to improve the win rate? Yes.

Let's start with doubling down. A double down situation is always advantageous - that's why you are doubling the bet. Wouldn't it be great if you could put out enough to triple your original bet? You can, if you're playing with funny money. Using the simplistic valuation of a funny money chip at 50% of its face value, putting a real money chip out for the double down results in a total that is three times the value of the original bet - a triple down!

The same principle applies to pair splitting, but with a difference. Some pair splits represent disadvantageous situations. These are called defensive splits. You split 8-8 vs. dealer 10, not because that will give you two good hands, but because 8-8 is so very bad that two mildly bad hands are preferable. You still expect to lose, but not as much as by hitting or standing. For such defensive splits, use funny money. Offensive splits (e.g., 8-8 vs. 6), when you expect to win both hands, are another matter. Now you want to get as much money on the table as possible. Use real money for the split (and resplit) hand(s), thereby getting twice the value of the original bet on the split hand(s). Together with the double downs, real money splits should get you over the 50% win rate.

What about insurance? If you could insure with funny money and win 2 for 1 in real money, that would be a 4 for 1 payoff! (4 for 1, not 4 to 1, because the dealer would take the funny money insurance bet.) You would almost always insure at that

payoff! In fact, it would theoretically be best to use funny money for insurance only. Unfortunately, the funny money usually comes with some fine print that says "even money bets only." You must use real money for insurance, but watch out - even then you may only get even money! I got in a battle about this at Vegas World. A real money insurance bet got me an even money payoff, despite the "INSURANCE PAYS 2 TO 1" printed on the felt. How can this be legal? I don't know, but you had better ask about the payoff before insuring. "Even money bets only" is also their excuse for paying 1 for 1 on a Blackjack, despite the fact that the 3 to 2 Blackjack payoff is what makes the game close to an even money proposition in the first place.

Since real money double downs and offensive splits change the relative merits of alternative actions, some strategy changes are in order. I suspect that even some hit/stand strategy changes are affected (a tie becomes a little more attractive, since you get to save the funny money for a possible double down or split). Perhaps 12 vs. dealer 4 is a basic funny money hit, going for win, tie, or lose instead of win or lose. I'll leave these considerations to the mathematicians.

Watch out for the Vegas World deal. They will give you \$500 in funny money, plus \$100 in "slot certificates," for \$296 in real money. The slot certificates must be played on special machines, whose payoff is extremely bad. Since a Funny Money blackjack will pay only 1 for 1, you might as well play their single deck "Experto 21," in which the whole deck is dealt out and Blackjack always pays even money anyway. A counter will get great benefit from the dealing depth, but the widest bet spread tolerated is 1 to 4. The only good thing I have to say about Vegas World is that they pay a 10% bonus on horse race winnings. And I don't play the horses.

[The above casino information is somewhat out of date, since the article appeared in the March 1984 issue of Arnold Snyder's great *Blackjack Forum* magazine. However, the recommendations for playing funny money are correct. Arnold gave my original submission to a Blackjack guru, math professor Peter Griffin, for checking, who at first said it was wrong. After reconsidering, he got back to Arnold and said I was right. Just imagine how much that pleased me.]

## **THE BLACKJACK COMBINE : COMPENSATION ISSUES FOR BLACKJACK TEAMS**

**By Marvin L. Master**

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Want to make money faster at blackjack? Join a blackjack team! A team or combine, is a group of investors/players who combine their individual bankrolls and/or playing talents. Each blackjack player can then legitimately size his bets according to the total team bankroll. For instance, four players with four identical bankrolls can combine their money and then bet four times their normal bets. Result: Each makes four times as much money, with no increase in risk. There are two provisos: (1) They must not play at the same table, and (2) the current total bankroll must be communicated frequently to the blackjack team players.

The "blackjack combine" concept differs from the standard "blackjack team" concept by a more equitable sharing of profits and losses. A combine member may be an investor, a player, or both. The players bet the combined bankroll per Kelly principles, keeping half of all winnings for themselves. If a player loses, he gets no money for playing until the loss is made up. Then the 50-50 split resumes. The second 50% of player winnings goes to the combine treasury. The treasury works just like a money market fund: Investors share profits (and losses) in proportion to their current shareholdings.

This idea works with one player with no bankroll teamed with one investor who doesn't play. If a plan doesn't work for this situation, then it's no good for more complex teams. Ken Uston's team plan (in *Million Dollar Blackjack*) does not work for this situation.

Uston's plan says that the 50% allocated to players should be divided 25% for time and 25% for money won. Moreover, a loss is not made up. This method is inequitable and unsound: (1) It is difficult to define and keep an accurate account of "time." Some players waste time in a casino; others are more efficient. Besides, equal time should not pay equal money to unequal players. Better to combine the time and skill elements, since those who work longer and play better will presumably win proportionately. That's good enough, and much simpler. (2) It would be very tempting for a player to delay reporting a win or time played when he knows that the group is losing. He would not get rewarded for his performance, so why not wait?

The key point here is that a win is not a "win" until any previous loss is made up. The play never "ends" until the combine is dissolved. It's a continuous series. If that series is broken at arbitrary points in order to distribute profits since the last point, the overall profit for each person will depend on when the breaks occur. To see that this is so, imagine that I had an investor who bankrolled me for monthly trips to Las Vegas for one year. Suppose I alternately lose \$500, win \$1000, lose \$500, win \$1000, etc., for the whole year. If we settle up monthly giving him 50% of every win and 100% of every loss, the investor ends up with nothing while I get \$3000. If we settle up at the end of the year instead, we each get 50% of the profit, \$1500 apiece. Using my method of not taking a win until a loss is made up, the payoffs are the same either way.

The bankroll "share" concept presents some interesting facets. Typically, 50% or so of

the "declared" bankroll should be liquid, readily accessible to the treasurer and players. This liquidity percentage depends on the number of persons who may be playing at the same time, and the readiness with which money can be passed back and forth. Because bet sizing is a personal matter (Kelly betting may be more, or less, aggressive than suits a person's circumstances), an investor with, say, a \$5000 bankroll may wish to "declare" anywhere from \$2500 to \$10,000. If the team is betting per Kelly then declaring only one-half of one's bankroll is effectively betting half-Kelly. This policy will lower the probably win but will substantially reduce fluctuations and increase the probability of being ahead at any given time.

Conversely, the \$5000 investor may want to declare in at the maximum for his holdings, or \$10,000. His \$5000 will satisfy the 50% requirement for a \$10,000 declaration. Perhaps he has an alternative source of ongoing income that may be considered bankroll. Or he might want to make a lot of money fast, and be willing to risk the 50% chance of being a loser (when betting Kelly x 2) in order to have the chance at a big killing.

The treasurer must maintain a separate account for investors' funds that are not part of the official combine bankroll, to be used for adjusting shareholding as instructed by the investors. A person may say, "I want all wins put aside for me. Keep my bankroll value constant." Perhaps he needs the income now. Those who are declaring more or less than their real bankroll will give instructions to the treasurer to adjust "declaration" bankroll after a team win or loss according to their particular circumstances. The half-Kelly investor will want this share reduced by only  $\frac{1}{2}$  of any loss, and increased by just  $\frac{1}{2}$  of any win. His separate account will be used for such adjustments. The Kelly x 2 investor wants his share increased by twice any gain and decreased by twice any loss. He will not need a side account, because his 50% requirement is automatically maintained by this policy.

Expenses, being a personal matter, should come out of a person's own money. If a player has no money for expenses, the treasury can advance the required amount, to be repaid out of the player's share of his winnings.

You can have a "team," or numerous "teams" within a single combine. A "team" differs from a combine in that team members play *as a team*, working together in a casino. The usual arrangement is for one team member to be a "big player," placing large bets at various tables as secretly directed by low-betting teammates who are scattered around the casino doing the counting. A blackjack team can thus be part of a combine but in this case the team would be treated, by the rules of the combine, *as a single player*. The treasury gets 50% of any wins, with the remaining 50% split among the team members in accordance with the team's own rules. The big player, for instance, may be a highly skilled person to whom the team votes more than a normal share.

## **INSURE A GOOD BLACKJACK HAND? PART I**

**By Marvin L. French (aka Marvin L. Master)**

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Should you insure a good blackjack hand? Blackjack gurus ridicule this question, replying that insurance is a side bet that has nothing to do with the player's hand. If more than one-third of the unseen cards are ten-valued then you insure; if fewer, you don't.

But what if the tens make up exactly one-third of the unseen cards? That makes the 2 to 1 insurance payoff exactly right, with no advantage to the casino or blackjack player. At first glance it seems that taking insurance in this case is wrong. It's like taking the odds in craps; you increase your bankroll fluctuations without any long run gain.

But wait. Let's look at the statement that the insurance bet has nothing to do with the original bet. This is not true, because correlation is involved. If you have a natural, the correlation is perfectly negative, -1.0. Whichever bet wins, the other loses. If you do not have a natural but the dealer does, then the negative correlation is also perfect: You lose the original bet and collect on the insurance. But what if neither you nor the dealer has a natural? Now the correlation between the lost insurance bet and the result of the original bet depends on the quality of your hand. If you have a 20, the correlation will be highly negative: The insurance bet is lost, and the original bet will probably win. With a 16, however, the correlation will be positive: The insurance bet loses, and the original bet will probably lose too.

These correlations lead to some interesting conclusions when there are exactly one-third tens in the deck. If you have a natural, then taking insurance should be automatic. It costs you nothing in the long run, and reduces bankroll fluctuation. If you have a 20, it seems to me that the decision should be the same. You will probably win the hand if you lose the insurance, so insuring to reduce fluctuation seems like a good idea. With a 16, however, bankroll fluctuation is increased, not decreased, by the fair insurance bet. I speculate that a player hand of 11, 19, or 20 should take the fair insurance bet, but other blackjack hands should not. Do any mathematicians out there care to comment?

## **INSURE A GOOD BLACKJACK HAND? PART II**

**By Peter A. Griffin (Professor of Mathematics and Statistics, California State University (CSU), Sacramento, CA)**

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[Ed. note: In the December issue of *Blackjack Forum* (Vol. VII #4), Marvin L. Master conjectured that if your card counting system indicated that the insurance bet was dead even, it may be advisable to insure a "good" hand, since this play would tend to reduce fluctuation. Marvin's logic is clear. If the dealer does have a blackjack, then you will lose a bet you expected to win. Taking insurance would save this bet on one third of these hands, and on those hands where the insurance bet loses, you still expect to win your

initial "good" hand. Thus, bankroll fluctuations are reduced.

Here now, to lay this to rest, is Peter Griffin's final word on whether and when you should take insurance on "good" blackjack hands. More probably, this article will give nightmares to players who consider attempting to work out Griffin's insurance formula when playing. Griffin shows that it is sometimes advisable to insure good hands-in order to reduce fluctuations-even when the insurance bet has a negative expectation! Unfortunately, most dealers only allow a couple of seconds for the insurance decision. So, the simplest answer is: Marvin was right! Insure your good hands when it's a dead even bet. --*Arnold Snyder*]

[The following is by Griffin, speaking of himself in the third person]

Marvin L. Master asks the question: Should you, to reduce fluctuations, insure a good hand when precisely one third of the unplayed cards are tens?

The answer depends upon what criterion for "reducing fluctuations" has been adopted. Griffin, in his monumental epic *The Theory of Blackjack* (Huntington Press, 1988), shows that there are occasions when a Kelly proportional bettor would insure a natural with less than one third of the unplayed cards being tens. Theoretically, this criterion could also be used to analyze whether to insure 20 and other favorable holdings. However, the answer is very dependent upon both the fraction of capital bet and the distribution of the non-tens remaining in the deck.

An approximate calculation based upon what would seem a reasonable assumption in this regard suggested that 20 should be insured, but 19 not. Precise probabilities for the dealer were not computed, and the answer could well change if they were, or if a different fraction than assumed were wagered.

Another, more tractable, principle to reduce fluctuations also appears in *The Theory of Blackjack*: When confronted with two courses of action with identical expectations (the insurance bet here is hypothesized to neither increase nor decrease expectation), prefer that one which reduces the variance, hence average square, of the result.

This proves particularly easy to apply here. Let W, L and T stand for the probabilities of winning, losing, and tying the hand assuming insurance is not taken. In this case the average squared result is

$$E_{N^2} = 1 - T$$

If insurance is taken the average square becomes

$$E_{I^2} = 1/3 \cdot 0^2 + W(1/2)^2 + T(-1/2)^2 + (L-1/3)(-3/2)^2 = (W + T + 9L - 3)/4$$

Insurance will have a smaller average square if

$$W + T + 9L - 3 < 4 - 4T$$

Equivalently

$$W + 5T + 9L < 7$$

Or, subtracting

$$5(W + T + L) = 5$$

$$4L - 4W < 2$$

$$L - W < .5$$

$$L < W + .5$$

This will clearly be the case for player totals of 20, 19, 18, 11, 10, 9 and 8 if the dealer stands on soft 17. If the dealer hits soft 17, 18 would probably still be insurable, but not 8.

Returning to the Kelly criterion, the interested reader would be well advised to consult Joel Friedman's "Risk-Averse" card counting and basic strategy modifications. Among Joel's astute observations is that if a player confronts an absolute pick 'em hit-stand decision he should hit rather than stand. The reason is that he thereby trades an equal number of wins, (+1)2, and losses, (-1)2, for pushes, (0)2, thus reducing fluctuation.

[Note: Peter Griffin died after a short bout with cancer October 18, 1998.]

## CONTROLLING THE TABLE

By "controlling the table" I mean getting more good hands than bad hands, more good hands than any table companion(s), extracting more cards per deal when the odds favor the house, and playing last with tablemates in order to see as many played cards as possible before making a play decision, all the while putting on an act that makes you look like a stupid gambler.

### **Playing One-on-One**

When for some reason (perhaps to justify "comps") your minimum bet must be twice the table minimum (\$20 at a \$10-table, \$50 at a \$50 table, etc.) use the following strategy, which probably won't get you barred. When the odds are in your favor, play one hand. When the odds favor the house, divide your minimum bet and play two hands. At a \$10 table a \$50 minimum bet (but two \$25s when the count is in the house's favor) should get you some comps. You can let a win "ride" without much suspicion, but to be safe you might avoid increasing the bet otherwise. Of course the minimum bet can be higher if you can risk it, say \$100 for a single hand at a \$25 table and two \$50s for two hands, and then the comps should be pretty good.

Splitting your minimum one-hand bet in order to play two hands when the count is bad has a number of advantages: (1) there are 50% more cards eliminated from a bad deck residue than when playing a single hand; (2) you get some advantage in the play of the second hand by having seen your first hand's cards before making a playing decision (and you can look at both hands before making an insurance decision); (3) playing two hands reduces the fluctuation of bankroll to some degree (the variance is smaller); and (4) a pit boss may note that you spread to two hands when the count looks negative to him. Since going to two hands is usually a bullish-looking action, he probably won't be suspicious.

If the count has gone well plus, you are likely to have lost a hand or two to those small cards that the dealer got previously, so you have an excuse for going to one hand if you need one: "I need to change the order of these cards!"

If you win one hand and lose the other, the odds now good, just let the win ride on its spot and abandon the losing spot. If you have won both bets, just pile everything on one spot, how could that look suspicious?

In sum, for the same money on the table per deal, you get more deals playing one spot and fewer deals playing two spots.

When it seems that you are going to get only one more deal before the shuffle, then going to two hands is okay when the odds favor you. That sometimes gets you a shuffle, however. You might ask the dealer, "Do you have enough left for me to play two hands?" Of course then he might say no and shuffle!

As I said, even without increasing the total money wagered, playing two hands instead of

one tends to reduce bankroll fluctuation by reducing the variance of the final result. Does that need explaining? Betting \$100 on one hand or \$50 on each of two hands have close to the same expectation (close, because you get to see one hand before playing the other), but the former bet will have a standard deviation that is greater, and bankroll fluctuation is something to be minimized.

If not worried about getting barred, you can use a different strategy. When the amount you would like to bet on one hand is suspiciously large, reduce it a bit on the one spot and play another with the same amount. That means using a strategy opposite to the one suggested above. When the odds are in the house favor, play as little as you dare on one spot. That's a good way to get barred, in my experience, so you need a good "act" to get away with it.

### **Playing with One or More Tablemates**

As always, try to sit at the far left of anyone else at the table, so that you get to see more played cards to aid your playing decisions. Play just one hand when the odds are in the house's favor and two hands when the odds are in your favor, thereby getting more good hands than the person playing a single spot. With one other at the table who is playing a single spot, you get two-thirds of the good hands and he gets only one-third, obviously beneficial to you. With two companions you get one-half of the good hands when playing two spots, and only one-third of the bad hands when playing a single spot. Encouraging others to play two hands when the odds are bad isn't nice, but who cares? They're going to lose their money anyway, eventually.

Another plus is that if you are playing the table minimum of, say \$25, you must put out twice that on each of two hands, resulting in four times as much money on the table as when playing one hand. This strategy may mean you need a pretty good act to avoid getting barred, but going from one green (\$25) to a mandatory four greens doesn't look too bad at a \$25 table. However, a \$100 minimum table gets watched pretty closely, requiring a very good "act" when using this strategy.

When doing all this, you must be careful about getting fewer rounds because of playing two hands when the odds favor you in a hand-dealt game. If a single-deck dealer is dealing three rounds to four players, playing two hands on the second round may cut that to two rounds. It is better to play single bets on two rounds than two bets on one round, as the second single spot's bet and play will be made on the basis of updated information. However, it is usually possible to play two hands on the last of three rounds, especially if you wait until the last moment to do so.

### **Camouflage**

For those who don't know, an "act" consists of convincing the management that you are a typical gambling bozo. Douse yourself with bourbon before playing, bring a glass of ice water with an olive in it to the table to look like you're drinking a martini, get a pretty gal to hang on you while you play and occasionally scrounge chips from you, smile and laugh

a lot, and so on. Such "camouflage" acting is an art that high rollers must adopt if they want to play very long.

### **Card Eating**

As some of the above tactics accomplish, you want to get as many cards on the table as possible when the odds favor the house, in order to minimize the number of hands you must play from that deck remainder. This is known as "card eating." One card-eating tactic is to hit a hand that is already busted. If you have been asking for dealer help in adding up your hand from time to time, as you should in order to appear stupid, you can get away with this-- but don't do it too often.

### **Tipping**

Don't do it. I have seen players put out a token (casino language for a tip put out in front of your bet as a bet for the dealer) in order to prevent a shuffle when the count is good, but the amount of the token is greater than the expected return on the bet! A \$5 token on a \$50 bet doesn't seem like a lot, but that's 10% of the bet and expected return on a bet is rarely that good.

If you must give tokens, give the dealer 1/2 of a dealer win and keep the other half for a token on the next hand. If you win five in a row, a \$5 token will earn the dealers (they share) \$25 and get you some good will. If the house doesn't allow that (some don't) put your token on top of your bet, but offset a little. The dealer will understand what you are doing, and no one can tell you what to do with that sort of token. Just don't forget to give him his piece of a win, as forgetting will not be forgiven. You might want to token when leaving the table after a big win if you must play frequently in the same casino, but never after a loss and never if you are not going to be remembered. Dealers do make most of their money from tokens, but this is a tough game in which no quarter should be given. :))

# Optimal Betting

Marvin L. French

You have just calculated that the San Diego Chargers are 60% sure to beat the Oakland Raiders in an upcoming playoff game. The sports book shows the game even, so you don't have to give up any points if you bet on either team. You must give odds, however, since the sports book pays only 10 to 11 on winning bets. You have a \$10,000 betting bankroll. Now you ask yourself, "How much should I bet on this game?" If your answer is about \$1600, you may want to skip the rest of this article. Otherwise, read on.

Betting on the outcome of a game is a game in itself. We call such a game favorable if we are more likely to win money than to lose money, so a bet on the Chargers appears to be a favorable game. We'll have to check on that, however, since odds are involved. Those who bet on the Raiders are in an unfavorable game, if the 60% Charger win probability is correct. The sports book handicapper believes the Charger-Raider match is a "fair" game, in which neither side has an advantage. If the teams are really evenly matched, the 10 to 11 payoff makes the betting game unfavorable to bettors on both sides. That is how a sports book makes money. If you were to make such a bet with a friend at even money odds, however, you would be in a favorable betting game.

Game probabilities are usually expressed in terms of  $p$ ,  $q$ , and  $r$ . The letter  $p$  stands for the probability of success, with  $p = 0$  if there is no chance of winning and  $p = 1.0$  when a win is certain. If the Chargers are 60% sure to beat the Raiders, then  $p = .60$ . The letter  $q$  represents the probability of failure, which also has a range of 0 to 1.0. If there is no chance of a tie, true in this case, then  $q = .40$  for the Charger bet. The letter  $r$  designates the probability of a tie. If the Charger-Raider game were a regular season contest, a tie would be possible. If there is a 2% chance of a tie, then  $r = .02$ . Since  $p + q + r$  must total 1.0, the probabilities for winning, losing, and tying with a bet on the Chargers would then be  $p = .60$ ,  $q = .38$ , and  $r = .02$ , if we say that the Chargers' chance of winning is still 60%.

To complete the description of the betting game we need to state the payoff for a win compared to the loss "payout," or in other words, the odds offered for the wager. For these we use the Greek letters  $\alpha$  and  $\beta$ ,  $\alpha$  standing for the win payoff and  $\beta$  for the loss payout (the total wager). The ratio of  $\alpha$  to  $\beta$  tells what the betting odds are. For a football wager in a sports book, the odds offered are generally 10 to 11, so  $\alpha = 10$  and  $\beta = 11$ . To win 10, you must risk 11. We now have values for all the parameters governing a bet on the Chargers in the playoff game:  $p = .60$ ,  $q = .40$ ,  $r = 0$ ,  $\alpha = 10$ , and  $\beta = 11$ .

## Expected Gain

How much is this bet worth? We'll use the term "expected gain" for this purpose. Expected gain is not the same as "edge," however. That comes next. Expected gain ( $E$ ) is the payoff ( $\alpha$ ) times the win probability ( $p$ ), minus the payout ( $\beta$ ) times the loss probability ( $q$ ):

$$E = \alpha p - \beta q$$

For our football game, the calculation is  $E = (10)(.60) - (11)(.40) = 1.6$ . You are risking 11 to win 10, with a 60% chance of success and a 40% chance of failure. Eleven what? Ten what? It doesn't matter. Whether they are one-dollar bills or hundred-dollar bills, you may expect to win 1.6 of them. Let's call them units. "Expect" means that if you were to bet a huge number of games identical to this one, the average of all your wins and losses would be very close to 1.6 units per game. You can't win 1.6 units on one game, of course, just as you can't have an average number of children (1.7). You either win 10 or lose 11, but the expected gain is 1.6.

When solving the above equation, if  $E$  turns out to be zero or a negative value, you don't place a bet unless you're a "gottabet" who must bet every game. Suppose the Chargers had only a slightly better than even chance of winning, say 52%. Then  $p = .52$ ,  $q = .48$ , and  $E = (10)(.52) - (11)(.48) = -.08$ . The expected gain turns out to be an expected loss, so you skip this game. You need to have  $p$  a little larger to consider a bet at the odds offered, and a lot larger to make a bet really worthwhile. For a 10 to 11 payoff, the expected gain is close to zero when  $p = .524$  and there is no tie possibility.

## The Edge

Now, is 1.6 the "edge" (advantage) for the Charger bet as originally stated? Obviously not, although many writers on gambling have the habit of treating edge as synonymous with expected gain. No, the edge, or advantage ( $A$ ), is the ratio of expected gain ( $E$ ) over amount risked ( $\beta$ ):

$$A = E/\beta = (\alpha p - \beta q) / \beta$$

We have calculated that  $\alpha p - \beta q$ , the expected gain, is 1.6 for the Charger bet, so the advantage is  $1.6/11$ , or .1455. In percentage terms this is a trifle more than a 14.5% edge. For every dollar you risk on this football game, you may expect to win 14.5 cents. Here too, "expect" means an anticipated average result if the same bet could be made on identical games a huge number of times.

For even-money wagers, as when betting with a friend, the win and loss amounts are equal ( $\alpha = \beta$ ) and the advantage equals the expected gain for a one-unit bet. Both would have a value of  $p - q$ , which for the Charger bet is  $.60 - .40 = .20$ . Thus, the friendly wager on the Chargers would have a 20% edge instead of 14.5%, the difference representing the sports book's share of the action.

## Bankroll Growth Rate

All right, so you're betting 11 units to win 10, but you want to know how large the units should be in terms of dollars. The question is not immediately answerable, because the appropriate amount depends on your personal financial goals at the moment. Do you need a lot of money fast? Then perhaps you should bet the entire \$10,000 bankroll. That would put you out of business with a loss, however. Maybe you would be content to have a 95% chance of not going broke in the course of making 20 bets identical to this one. Then you must bet a smaller amount, about \$1350.

Instead of one of these rather subjective aims, let's assume that your goal is more objective: To maximize your bankroll's probable rate of increase over the long haul, with a minuscule chance of ever going broke.

Bankroll growth rate, or rate of return, is analogous to a compound interest rate. If you have a bank account that carries a 6% interest rate, compounded annually, then the account will grow by a factor of 1.06 each year. After two years the total will equal the original deposit times 1.06 times another 1.06, which comes to 1.1236. To show the growth factor after many years we use an exponent instead of writing 1.06 umpty-ump times. For instance, after ten years the account would grow by a factor of  $1.06^{10}$ , about 1.79 times the original amount. The .06 value is the annual rate of return, and 1.79 is the growth factor after ten years.

It is intuitively obvious, and true, that the only way to get a healthy bankroll growth rate with safety is to always bet a fraction of current bankroll that is related to the "favorability" of the wager. We call this policy "fractional betting." No other betting strategy, no progressive betting scheme, no flat bet philosophy, can match the expected bankroll growth rate achievable with fractional betting.

## Optimal Bet Size

But what fractions of current bankroll are proper for advantageous bets? Assuming that you will have average luck (the number of wins and losses will equal expected values), then you probably want fractions that will result in a maximum rate of return for average luck.

Rate of return is normally thought of as a rate per unit of time (per year, in the bank account example). In a gaming context, however, it's a rate of return per play of a game. If the win/loss ratio is the expected  $p/q$ , the equation for the expected rate of return ( $R$ ) is:

$$R = (1 + \alpha f)^p (1 - \beta f)^q - 1,$$

where  $f$  is the fraction of bankroll represented by one betting unit.

For the Charger wager you are betting 11 such units, with  $p = .6$ ,  $q = .4$ ,  $\alpha = 10$ ,  $\beta = 11$ . You want to choose a fraction  $f$  that will result in a maximum value for  $R$ . This optimal fraction will be designated as  $f^*$ , which can be derived from the last equation by any first-year calculus student:

$$f^* = p/\beta - q/\alpha$$

This is equivalent to  $f^* = (\alpha p - \beta q) / \alpha \beta = E/\alpha \beta$ , the expected gain divided by the product of the odds. Since the advantage  $A$  is equal to  $(\alpha p - \beta q) / \beta$ , we can also say that the optimal fraction is equal to the advantage divided by the payoff:  $f^* = A/\alpha$ . You will sometimes read that  $f^*$  is equal to the edge, but that is true only when the odds are even ( $\alpha = \beta$ ) and there are no tie possibilities. In such a game, the values of  $f^*$ ,  $E$ , and  $A$  are all equal to  $p - q$ .

The edge for the Charger bet is .145 and the payoff is 10, so the optimal fraction ( $f^*$ ) of bankroll for a betting unit in this game is  $.145/10 = .0145$ . You are betting 11 times this fraction ( $\beta = 11$ ), so the

optimal wager is  $(.0145)(11)(\$10,000)$ , about \$1600. If you were to bet on this game with a friend at even odds, the value of  $f^*$  would be  $p - q$ , which for  $p = .6$  and  $q = .4$  is  $.2$ , making the optimal bet \$2000. This is both the unit size and the bet size, since  $\alpha = \beta = 1$ .

Going back to the sports book bet, what rate of return for average luck can you expect from the \$1600 bet? To find out, just plug 0.0145 ( $f^*$ ) into the equation for expected rate of return:

$$R = [1 + (10)(.0145)]^{0.6} [1 - (11)(.0145)]^{0.4} - 1 = .0118$$

If you were to make this identical wager many times and figure out the rate of return that results after 60% wins and 40% losses, the value for  $R$  would be  $.0118$ . That makes the growth factor  $1.0118^n$  after  $n$  such wagers. Note that the order of wins and losses makes no difference. Their ratio, not their order, determines the final result. This may or may not be a comforting thought during a losing streak. Contrast this with progressive betting schemes (e.g., double the bet after every loss, cut back on a win), in which the order of wins and losses may be very important.

Are you wondering about the validity of the equation for rate of return? Try it with whole numbers instead of  $p$  and  $q$  for exponents. In ten games with  $p = .6$  and  $q = .4$ , you figure to win six times and lose four times, right? If so, the bankroll growth factor after ten games is  $[1 + (10)(.0145)]^6 [1 - (11)(.0145)]^4$ , which comes to 1.1245, bringing your bankroll of \$10,000 up to \$11,245. Doesn't seem like much, does it? However, try any other fraction of bankroll for the betting unit, replacing  $.0145$ . You will not find one that produces a greater rate of return for expected luck. That is why we call  $f^*$  the optimal fraction, and the strategy of betting  $f^*$  units "optimal betting." This betting policy is optimal under two criteria according to Leo Breiman (whose book on the subject I cannot find, so I got this second hand): (1) minimal expected time to achieve a fixed level of resources, and (2) maximal rate of increase of wealth.

To verify that the 1.1245 growth factor after ten games represents a rate of return ( $R$ ) of  $.0118$ , we use logarithms:

$$\log(1 + R) = (\log 1.1245)/10 = .011734$$

$1 + R$  is therefore the antilog of  $.011734$ , which is 1.0118, making  $R$  the number we were looking for:  $.0118$ .

Can we express the expected rate of return for optimal betting ( $R_{\max}$ ) in terms of  $p$  and  $q$ ? Certainly. We know already that  $f^*$  bets figure to produce  $R_{\max}$ :

$$R_{\max} = (1 + \alpha f^*)^p (1 - \beta f^*)^q - 1$$

Substituting  $(\alpha p - \beta q)/\alpha\beta$  for  $f^*$ , and using the fact that  $p + q$  is 1.0 in a game with no ties, a little algebra produces an equation for  $R_{\max}$  in terms of  $p$ ,  $q$ ,  $\alpha$ , and  $\beta$ :

$$R_{\max} = [p(\alpha + 1)]^p [q(\beta + 1)]^q / \alpha\beta$$

For even-money unit bets ( $\alpha = \beta = 1$ ) this simplifies to:

$$R_{\max} = (2p)^p (2q)^q - 1$$

Applying this equation to the "friendly" charger bet, we get  $R_{\max} = [(2)(.6)]^{0.6} [(2)(.4)]^{0.4} - 1 = .02$ . The resultant expected bankroll growth rate for this bet,  $1 + R_{\max}$ , is therefore 1.02. Mathematicians prefer to talk about maximizing the logarithm of  $1 + R_{\max}$ , which they call  $G$ . I guess this is because optimal betting is equivalent to having a logarithmic utility function (doubling one's bankroll is twice as likely as losing half of it). Anyway,  $G_{\max}$  is easily obtained from the previous equation:

$$G_{\max} = \log(1 + R_{\max}) = p \log 2p + q \log 2q$$

## Games with Ties

Now, how do ties (r not equal to zero) affect our calculations? For one thing the previous  $f^*$  equation won't work. We must first change the  $p$  and  $q$  values, dividing each by their sum:

$$p' = p / (p + q) \quad q' = q / (p + q)$$

For the Charger bet with 2% chance of a tie,  $p' = .60/.98 = .612$ , and  $q' = .38/.98 = .388$ . The optimal bankroll fraction ( $f^*$ ) for a betting unit when ties are possible is:

$$f^* = (\alpha p' - \beta q') / \alpha \beta$$

For the Charger sports book bet,  $f^* = [(10)(.612) - (11)(.388)] / 10 / 11 = .017$ . For the friendly wager, for which  $\alpha = \beta$ ,  $f^* = p' - q' = .612 - .388 = .224$ . Here are the resultant respective rates of return:

Sports book:  $R_{\max} = (1.017)^{0.6} (.983)^{0.38} - 1 = .0036$

Friend:  $R_{\max} = (1.224)^{0.6} (.776)^{0.38} - 1 = .0252$

It is obviously much better to bet with a (non-welshing) friend than with a sports book! Note that we use the original  $p$  and  $q$  values, not  $p'$  and  $q'$ , for exponents in the  $R_{\max}$  equation.

In terms of  $p$  and  $q$ , the equation for  $R_{\max}$  in an even-money payoff game with tie chances becomes:

$$R_{\max} = (2p)^p (2q)^q - 1$$

And for the mathematicians:

$$G_{\max} = \log(1 + R_{\max}) = p \log 2p + q \log 2q$$

## Optimal vs Flat Betting

"Wait," you may say, "How about flat betting \$1600 every time instead of betting a fraction of current bankroll? The edge is .1455, so after ten flat bets of \$1600 I expect to win \$1600 times .1455 times 10, which is \$2328. That gives me a total expected bankroll of \$12,328 for ten games instead of the \$11,245 you got by so-called optimal betting. Why not flat bet instead?"

Yes, but ten games is not the "long haul." Let's look at 100 games. Now flat betting \$1600 wins an expected \$23,280, resulting in a bankroll of \$33,280. This assumes you can play on credit, because there is a 15% chance you will be down \$10,000 or more during the 100 games. The optimal bettor's bankroll with expected luck is \$10,000 times  $1.0118^{100}$ , or \$32,320. Almost the same result as for flat betting, but with no chance of going broke (since only a fraction of bankroll is used for every bet).

Now go on for a second 100 games. Flat betting (with credit) gives you an expected win of \$46,560, for a total bankroll of \$56,560. Assuming the same luck, optimal betting yields \$10,000 times  $1.0118^{200}$ , which is \$104,456. Much better!

The principle here is analogous to compounded vs simple interest. A compounded rate is eventually going to get you more money, even if a simple interest rate is greater. Assume a game with a small edge, with repeated plays by both a flat bettor and an optimal bettor. The flat bettor makes the same bet as the optimal bettor on the first play (and figures to win the same), but thereafter he continues to make identical bets. The optimal bettor wagers a constant fraction ( $f^*$ ) of his current bankroll. The optimal bettor does worse for a while in a typical game, but after a sufficient number of plays (depending on  $p$ ,  $q$ , and  $r$  values) he passes the flat bettor. From then on his bankroll grows dramatically while that of the flat bettor lags further and further behind

## **Non-Optimal Bet Fractions**

What happens to a fractional bettor who bets more or less than the  $f^*$  fraction? With smaller bet fractions the probable growth factor for bankroll increases as the fraction increases, topping out when the fraction reaches  $f^*$ . For fractions greater than  $f^*$ , the probable growth factor starts decreasing with increasing bet fractions, becoming 1.0 (no growth) at or near  $2f^*$  bets. For fractions larger than  $2f^*$ , the probable growth factor is less than 1.0 (bankroll shrinking), approaching zero as a limit. When a fraction of bankroll is bet every time, it is theoretically impossible to actually reach zero (but the bankroll may become less than the size of a minimum permissible bet!).

To summarize: If you have the expected amount of luck (actual win/loss ratio is  $p/q$ ), your bankroll grows fastest with  $f^*$  fractional bets, stays about the same with bets of  $2f^*$ , and actually shrinks for a policy of betting more than  $2f^*$ . Another item of interest is that equal degrees of overbetting and underbetting on each side of  $f^*$  results in equal (reduced) bankroll growth. You can't make up for overbetting by some compensatory underbetting. They don't compensate, so far as probable growth factor is concerned.

Also interesting are the bankroll fluctuations that occur over time for variations in bet fractions. Although bets of  $f^*/2$  and  $1.5f^*$  result in the same expected bankroll growth rate (less than that for  $f^*$ ), the smaller fraction will see a much smaller fluctuation in bankroll along the way. Betting  $1.5f^*$  brings exhilarating/scary swings in bankroll size over time. Mixing  $f^*/2$  bets and  $1.5f^*$  bets would

produce fluctuations somewhere in between, so they do compensate somewhat in that regard (but not symmetrically).

With  $2f^*$  fractional bets the results for a large number of players will be wildly different, some making a fortune, others losing almost the entire bankroll. The median player in the ranking of results should have close to the same bankroll he started with, although his bankroll has probably fluctuated greatly along the way. Above  $2f^*$  the median player figures to lose money and the fluctuations become even greater.

## The Median Fallacy

The above analysis seems to say that a fractional bettor will lose money in a coin-tossing game if he wins the expected 50% of the time. Since the edge is zero in such a fair game,  $f^*$  is also zero. Any bet fraction exceeds  $2f^*$  and will lead to eventual shrinkage of bankroll if wins and losses balance. Look at the rate of return for expected luck when betting 10% of bankroll on every coin toss:

$$R = (1.1)^{0.5} (.9)^{0.5} - 1 = -.005$$

Even in a favorable game, the rate of return for average luck will be negative if the bet fraction is greater than  $2f^*$ . Suppose you risk 50% of your bankroll in the "friendly" Charger bet, for which  $f^*$  is .20, or 20%:

$$R = (1.5)^{0.6} (0.5)^{0.4} - 1 = -.0334$$

Amazing! The fractional better appears to lose money in favorable games if he continually bets more than the  $2f^*$  fraction. But hold on a minute! How could any betting scheme in a fair or favorable game lead to an expected loss? A basic theorem of gambling says that all betting methods have the same mathematical expectation in the long run. Come to think of it, something else looks odd. For the "friendly" Charger bet,  $f^*$  is 20% of bankroll and the edge is 20%, so we expect to win 0.2 times 0.2 times the current bankroll = .04 times current bankroll when we make this bet. But the  $R_{\max}$  we calculated for this game is only .02, half of .04. Do we have a fallacy here?

Yes and no. Saying that a fractional bettor's win rate (total winnings vs total wagers) is only half a flat bettor's win rate is true in a way, but only for the median result, not for the average result. It's the median fallacy.

Suppose you have two teams of 1001 players each, all with identical starting bankrolls, and all betting on the same favorable propositions over a period of time. Members of one team continually flat bet an amount equal to the original optimal bet. The other team bets the  $f^*$  fraction of current bankroll on every proposition. The median result for each team--that of the 501st person in ranking of results--will indeed show the median fractional bettor's win rate to be about one-half that of the median flat bettor's rate. In fact, the comparison gradually worsens for the fractional bettor as the number of plays increases. But if you add up all the 1001 results for each team, you will find that each team has made about the same win rate on total wagers. That is, total money won divided by total money bet will be the same for both teams. As stated before, no betting scheme can change that number.

You may have noticed my continual references to "return for average luck" or (the same thing) "return for expected luck." In the case of fractional betting, this is not the same as "return for expected win." Expectation may be defined as the average result for an arbitrarily large number of players. Some will have good luck, some bad, and some (very few) average luck. For flat bettors, who bet the same number of dollars each time, average luck yields an average win total. For fractional bettors, however, the result for average luck is not the average result!

Take a coin-tossing game. Suppose a fractional bettor and a flat bettor start out with the same bet, say 10% of bankroll. Thereafter the fractional bettor always bets 10% of current bankroll, while the flat bettor makes identical bets for each toss. With average luck, the fractional bettor figures to lose money and the flat bettor breaks even. With good luck, the fractional bettor's bankroll rises exponentially, while a flat bettor's bankroll-to-luck relationship continues linear with luck. Equal degrees of luck on both side of average are equally probable, so for fractional betting the average result is greater than the result for average luck. With very bad luck, the fractional bettor loses less than the flat bettor, who may well go broke along the way. With very good luck, the fractional bettor may win a bundle. With average luck, the flat bettor breaks even while the fractional bettor loses.

If you take *all* possible degrees of luck, compute the fractional bettor's win for each, multiply each by its probability, and add it all up, what do you get? Zero, of course, the same as for flat betting. It's a coin-tossing game, isn't it? Zero has to be the average (expected) result for any fair game, even though for fractional bettors it is not the result for average (expected) luck.

But what about the "long haul?" Isn't everyone's luck going to be the same after a jillion plays? In a sense, yes. Everyone's luck is going to average out close to expected luck, *percentagewise*, as the number of plays gets huge. In absolute terms, however, the probable difference between actual number of wins and the expected number increases with number of plays. At the same time, the probability of having exactly average luck decreases.

In a coin-tossing game, one standard deviation for 100 tosses is +/-5%. This means that 68% of players in such a game will win between 45 and 55 times out of 100, within five wins of the 50-win average. For 10,000 tosses, however, the standard deviation is much smaller, +/- .05%. But 0.5% of 10,000 is 50, so the spread of one standard deviation is 4,950 to 5,050 wins, ten times greater than for 100 tosses. Very few fractional bettors will have the average 5,000 wins, resulting in a negative rate of return. Of the rest, half will do better, half worse. Some of those who do better will do sensationally well. Those who do worse will never go broke, theoretically, because they are always betting a fraction of current bankroll. The overall average result, however, remains at the immutable expectation for coin-tossing: exactly zero.

Of course all this is only of academic interest, since there is no incentive for betting in a game that isn't favorable.

## **A Computer Simulation**

Is it best to assume we're going to have average luck and bet on that basis? Not necessarily. If assumption of average luck were the optimum course in life, we would never buy auto insurance or spare tires. Besides, while average luck is the most probable outcome of many wagers, it is very improbable when compared to the probability of having better or worse luck.

To pursue this further, let's examine the following game:  $p = .51$ ,  $q = .49$ ,  $r = 0$ , 1-1 odds ("even money."). The edge is 2% and  $f^*$  is the same, .02. I fed this game into a computer and recorded the results of 100 simulated players, each starting with a \$1000 bankroll and each playing the game 5000 times with  $f^*$  bets (2% of current bankroll). A random number generator determined the outcome of each play. I confess that I expected the average of all bankrolls, after 5000 bets, to have grown according to the equation for  $R_{\max}$ :

$$R_{\max} = [(1.02)^{.51} (.98)^{.49} - 1 = .0002$$

The average ending bankroll for all players, I thought, would show a bankroll growth of  $1.0002^{5000}$ , which is 2.718 times the starting bankroll, or \$2718. To my surprise, the average came to almost \$7000. Had I done something wrong? Then it came to me: Of course! The average result is not the result for average luck! It is true for flat betting, but not for fractional betting. Of the 100 simulated players, those with close to expected luck (2550 wins) did indeed have final bankrolls of around \$2700. About half the remaining players did worse, half better. The worst result was a final bankroll of \$44.15, the best a whopping \$78,297. You see what happens? The lucky ones pull the average up much more than the unlucky ones pull it down.

For 100 players flat betting \$20 (the fractional bettor's original bet), we don't need a computer. They will win close to  $100 \times \$20 \times .02 \times 5000 = \$200,000$ . That's an average of \$2000 per player, which added to the original bankroll of \$1000 comes to \$3000. Compare that to the \$7000 average of the optimal betting players. Quite a difference!

Variations of the computerized game were very enlightening. I next tried 2000 players instead of 100. The average final bankroll was \$7122. The reason for the increase is that there is a very small percentage of players who do extremely well. The larger number of players makes for a better representation of the lucky ones. The luckiest of the 2000 ended up with \$161,568, while the player with my sort of luck had only \$24 left. As in the case of 100 players, approximately one-fourth lost money.

That's a sobering thought, isn't it? With optimal betting, there was only a 75% chance of being ahead after 5000 plays of a game with a 2% edge. This is roughly equivalent to 5000 single-deck Blackjack bets with a "true count" of plus four. No wonder some Blackjack players get discouraged!

My next step was to vary bet fraction, going from .01 to .05 times current bankroll ( $f^*/2$  to  $2.5f^*$ ). I also varied the number of plays: 1000, 3000, and 5000. The results are in Table I. Note how the average final bankroll continues to rise with bet fraction. Amazingly (to me, anyway), this is true all the way up to betting 100% of bankroll with every play! The few who never lose win such a googol of money that they bring the average up high enough to overcome the fact that everyone else goes broke.

STARTING BANKROLL \$1000	Number of Plays		
CONSTANT BET FRACTION	1000	3000	5000
.01 ( $f^*/2$ )	\$1206 (648)	\$1830 (436)	\$2710 (313)
.02 ( $f^*$ )	\$1474 (778)	\$3163 (625)	\$7122 (494)
.03 ( $1.5f^*$ )	\$1799 (867)	\$5378 (770)	\$18331 (725)
.04 ( $2f^*$ )	\$2171 (1045)	\$11549 (998)	\$43832 (1004)
.05 ( $2.5f^*$ )	\$2557 (1147)	\$16424 (1217)	\$104818 (1259)

Table I. Average Final Bankroll, 2000 Players, 2% Edge  
(Number of Losers in Parentheses)

Now look at how the number of losers (in parentheses) changes with bet fraction and number of plays. Small bet fractions increase the probability of being ahead after a given number of plays. For fractions less than  $2f^*$ , the number of losers appears to approach zero as the number of plays goes to infinity. The decrease toward zero is faster for small fractions, slower for larger fractions.

At about  $2f^*$  the number of winners and losers stays about the same for any number of plays. No matter how long you play with  $2f^*$  bets, you have close to just a 50% chance of being ahead (or behind) at any time.

Above  $2f^*$ , the probability of being a loser increases in both directions. Higher fractions or more plays both bring more losers, until you reach the ultimate of an infinite number of players betting 100% of bankroll forever. At this point we have the strange result I mentioned before: Although the chance of being a winner is minimal after  $n$  plays ( $p^n$ ), the expected bankroll growth factor is maximal  $(2p)^n$ .

How did I get  $(2p)^n$ ? To calculate the growth of the average final bankroll for a jillion players, you take the advantage ( $A$ ) times the fraction ( $f$ ) of a unit, times number of units risked ( $\beta$ ) to get average bankroll growth per play. The growth rate is therefore  $1 + Af\beta$  for each play, and the growth factor after  $n$  plays is  $(1 + Af\beta)^n$ . For 5000 plays of the 2% edge game the expected growth factor for bankroll is  $1 + [(.02)(.02)(1)]^{5000}$ , which is 7.386, bringing the bankroll to \$7386. My computer program result of \$7122 is slightly less because even 2000 players are not enough to get an accurate average with Monte Carlo programming.

Anyway, if the wager size is the entire current bankroll, the expected bankroll growth factor after  $n$  consecutive winning plays is  $(1 + A)^n$ , in this case  $1.02^{5000}$ , or  $(2p)^n$ . Put another way, there is a  $.51^n$

chance of not busting after  $n$  consecutive plays. That is  $p^n$ . If you haven't busted, your bankroll growth factor is  $2^n$ . The expected bankroll growth factor is therefore  $p^n$  times  $2^n$ , or  $(2p)^n$ .

Getting back to Table I, its lessons look rather paradoxical:

-- Betting less than  $f^*$  units reduces expected winnings, but increases the probability of being a winner.

-- Betting greater than  $f^*$  units increases expected winnings, but also increases the probability of being a loser.

-- With  $2f^*$  units, the chances of being a winner at any time are about 50-50, but expected winnings are greater yet.

-- With bets greater than  $2f^*$ , the probability of being a loser approaches certainty as the number of plays increases, although expected winnings become astronomical in size.

### **What's Best for You?**

How can we apply these conclusions in a practical way? The answer is a subjective, personal decision. If you want to increase your chances of being a winner at any time, you bet less than  $f^*$  units, perhaps  $f^*/2$ . You will be more sure of making money than with  $f^*$  units, but the winnings may not be great. If you want to increase those probable winnings, you bet more than  $f^*$  units, perhaps  $1.5f^*$ . You will be less sure of winning, but you could win a lot. Finally, there is the middle road of  $f^*$  unit bets, with an attractive combination of best probable bankroll growth rate and a good chance of being a winner.

How much of a chance? With  $f^*$  unit bets, there is a 67% probability of doubling your money before losing half of it. This compares with 89% for  $f^*/2$  bets and 56% for  $1.5f^*$  bets. You must pick what feels right for you. To what degree are you willing to sacrifice probable income in order to gain safety, or *vice versa*? To some degree the answer will probably depend on your bankroll size. A large bankroll may provide quite a comfortable income with a mere  $f^*/2$  unit bets, while a small bankroll owner might be tempted to wager with  $1.5f^*$  bets until his bankroll grows considerably. Maybe he can't live on the safer income of  $f^*$  units. If it doesn't work out, he will just have to get a job. It follows that your betting practices may vary with bankroll size, according to your wants and needs.

### **Optimal vs Optimum**

What is this word "optimal"? Why not "optimum"? Because I'm trying to make a distinction. An *optimal* bet fraction ( $f^*$ ) for a betting unit will bring the greatest rate of return if the ratio of wins and losses is the expected  $p/q$ . An *optimum* fraction is the one that brings the greatest rate of return ( $R_A$ ) for the actual win/loss ( $W/L$ ) ratio, which may not be  $p/q$ :

$$R_A = (1 + f)^W(1 - f)^L - 1$$

For the 2% edge game, you expect 510 wins and 490 losses in 1000 plays, and the *optimal* fraction ( $f^*$ ) is .02. If you actually experience 505 wins and 495 losses, the *optimum* fraction would have been .01, which is a conservative  $f^*/2$ , with a rate of return of:

$$R_A = (1.01)^{505}(0.99)^{495} - 1 = .051$$

An optimal bet of .02 times bankroll would have produced a zero return:

$$R_A = (1.02)^{505}(0.98)^{495} - 1 = 0$$

In the other direction, the optimum fraction for better than average luck will be greater than  $f^*$ . For 515 wins and 485 losses, the optimum bet is 0.3 times bankroll ( $1.5f^*$ ):

$$R_A = (1.03)^{505}(0.97)^{495} - 1 = .568$$

Compare this to an optimal  $f^*$  bet:

$$R_A = (1.02)^{515}(0.98)^{485} - 1 = .492$$

Since we can't know what the optimum is going to be, we choose the most likely ratio for W/L, which is  $p/q$ . A bet fraction based on  $p$  and  $q$  ( $f^*$ ) will be optimal, but it probably won't be optimum.

## Alternative Wagers

Suppose you are offered a choice among various favorable wagers. You are allowed to make one bet only. Is it easy to choose? Yes, if all potential wagers are offered at the same odds and the same-size bet. You naturally pick the one that has the best chance of winning. But what if the odds offered are different, the win/loss probabilities are different, and/or there is a specified bet size, perhaps different for each? Let's take an example:

A friend offers you a choice of two wagers: Game 1 for \$1000 at even money, Game 2 for \$1500 and you get odds of 3 to 2. You calculate that Game 1 is 10% in your favor:  $p = .55$ ,  $q = .45$ , with an expected gain of \$100. Game 2 is 14% against winning,  $p = .43$ ,  $q = .57$ , but you are getting 3 to 2 odds, so your expected gain is \$112.50. Dividing this by the amount risked, \$1500, gives an edge of 7.55%. Game 1 has the better edge, but Game 2 has a greater expected gain. Which bet should you take with your \$10,000 bankroll? To find out, let's look at the rate of return for each.

For Game 1  $f^*$  is  $.55 - .45 = 0.1$ , so the \$1000 is an  $f^*$  bet. The expected rate of return ( $R_{\max}$ ) is  $(1.1)^{.55}(0.9)^{.45} = .005$ . For Game 2  $f^*$  must be calculated using the odds:

$$f^* = [(3)(.43) - (2)(.57)] / (3)(2) = .025$$

Betting two  $f^*$  units to win three  $f^*$  units would bring an expected rate of return of:

$$R_{\max} = [(1 + 3(.025))^{.43} [1 - 2(.025)]^{.57} - 1 = .0119$$

With the stipulated bet of \$1500, however,  $f$  is  $1500/10000/2 = .075$  ( $3f^*$ !), and the expected rate of return brings bankroll shrinkage:

$$R = [(1 + 3(.075))^{.43} [1 - 2(.075)]^{.57} - 1 = -.0054$$

You bet on Game 1, because not only is the expected rate of return better, but the fraction of bankroll for a betting unit is  $f^*$ , making this bet much safer. Those who choose a bet solely on the basis of expected gain may prefer Game 2. That would be a mistake in this case, if one wants to be an optimal bettor.

A richer person might rightly choose Game 2. For a \$30,000 bankroll, Game 2 represents an  $f^*$  bet ( $.025 \times 30,000 \times 2 = 1500$ ). The rate of return, already calculated, is .0119. The \$1000 Game 1 bet would be only 1/30 of this bankroll, giving a rate of return of  $R = (1 + .0333)^{.55} (1 - .0333)^{.45} - 1 = .0028$ . This 1/3  $f^*$  bet would be overly conservative for most gamblers, who would choose the alternative wager (an  $f^*$  bet).

When alternative wagers have equal rates of return, you choose the one for which the required bet represents a fraction of bankroll vs  $f^*$  that is closer to your betting philosophy. Suppose two wagers have an expected rate of return of .08, but one requires a bankroll fraction bet of  $0.8f^*$  units, the other  $1.2f^*$  units. You then have a choice between a conservative bet and an aggressive bet. It's your decision, no one can make it for you.

When the expected rates of return for alternative wagers are not equal, you usually pick the one that has the greater rate of return, but not necessarily. If the risk factor (in terms of  $f/f^*$ ) for the better rate of return is unacceptably high, you may go for the safer bet. Again, it's a subjective decision. If  $f/f^*$  is too high for both wagers, you might decide not to bet at all.

## Forced Wagers

There are times when we are forced to select between two wagers, both of which are unfavorable. Forced? Yes. If you own a house, you *must* choose between two bets. One is with the insurance company. You can annually bet a certain  $\beta$  amount (the premium) that your house will burn down. The insurance company will take the bet, offering huge odds on the proposition that the house will not burn down. We can expect a negative expectation, since insurance companies make money.

Alternatively, you can bet with Lady Luck, taking the "not burn" side of the proposition by not insuring. Since  $\alpha = 0$  (there is no payoff for a win), this also has a negative expectation. If you can calculate the chances of your house burning down, just plug that number into the equation for rate of return, along with all the other numbers, and choose the wager that brings the better (less negative) rate of return. In this case, "bankroll" is your net worth. If your house value is at all commensurate with that net worth, insurance is probably the better bet.

## Compound Games

In a compound game you must make some minimum bet on each game in a series, even though the expectation is frequently negative, zero, or so small that  $f^*$  represents a bet that is less than the allowed bet. However, sometimes the expected gain for a game in the series is significant, and you are allowed to make larger bets than the minimum.

Now what? You want to play because big bets on position expectation plays will bring good results overall, but you know that those bets which are greater than  $f^*$  compromise the reasonable safety that goes with constant  $f^*$  bets. To achieve the safety that  $f^*$  units provide, you will have to reduce your bets to something less than optimal when expectation is positive, in order to offset the probable reduction in bankroll caused by bets made with poor expectation. The amount of bet reduction will depend on the degree of overbetting imposed by the nature of the particular compound game, and the player's readiness to abort a game that turns negative.

## Multiple Payoff Games

Now for multiple payoff games, in which you may win or lose different multiples of your original bet. The expected rate of return ( $R$ ) for an original bet of bankroll fraction ( $f$ ) that will result in winning or losing various multiples ( $m$ ) of  $f$  with various probabilities ( $p$ ) is:

$$R = (1 + fm_1)^{p_1} (1 + fm_2)^{p_2} \dots (1 + fm_n)^{p_n} - 1$$

Some  $m$ 's are negative, and the  $p$ 's total 1.0

Finding  $f^*$  is far from trivial here, involving calculus that is beyond my meager knowledge, but there is a shortcut that works well if the overall advantage is small--as is usually the case in the real world. Just calculate the variance of all the possible results for a one-unit bet, and divide the overall advantage by the variance. The result will be a bet fraction that is close to the theoretically correct  $f^*$ . To get the variance, calculate the average of the squares of each possible result and subtract the square of the average result, with each result weighted by its probability.

## Complex Games

A compound game with multiple payoffs per play is a complex game. The prime example is the casino game of 21, popularly called Blackjack. ("Blackjack" is really the home-type game, in which the participants take turns as dealer.) When the proportion of small cards dealt shows that the remaining cards favor the player, an optimal wager is in order. The methods for estimating the advantage are best described in *The Theory of Blackjack* by Peter A. Griffin.

Many players (and writers) use this advantage percentage as a bankroll proportion for an optimal bet. The proper approach is to divide the advantage by the variance to obtain an approximately correct  $f^*$ . The variance for various 21 games in most situations is in the range of 1.3 to 1.4, so ignoring this factor can lead to some significant overbetting for the optimal bettor.

Since 21 is a compound game, it is wise to reduce bets even further to compensate for the excessive bets that are unavoidable for most styles of play. Two-thirds of the time the cards do not favor the player, so a “sit-down” player is overbetting his bankroll most of the time, even with minimum bets. The “table-hopper” (who leaves when the “count” goes negative) does better in avoiding these situations, while the “back-counter” (who lurks behind the table, waiting for a good count) never overbets. Each will have a different reduction factor for their optimal bets, which will also take into account the following inefficiencies:

1. The calculation of the advantage is necessarily approximate.
2. Bets must be rounded off to chip values.
3. Radical bet changes may not be tolerated by the “pit boss.”

## Twenty-One

The casino game of Twenty-One, popularly called Blackjack (actually the home version of the game), achieved wider popularity when computers showed in the early 60s that the game could be beaten. Players flocked to Las Vegas and Reno, thinking to win scads of money, but the great majority did not have the skills and discipline needed to win. The casinos loved the extra business. The film "21," based on the book by Ben Mezrich entitled "*21: Bringing Down the House*," screen play by Peter Seinfeld and Allan Loeb, tells the story of a group of students from the Massachusetts Institute of Technology (MIT), who in 1994 formed a team of players with the initial goal of beating the game in Las Vegas casinos. The film's hero was named Ben, the book's was Kevin. This was not a new idea, as professional player Ken Uston had done the same a decade before. The MIT team merely copied his tactics, with minor variations. Before going further, let's look first at a 21 table and typical rules for its game:

Four or more half-moon shaped 21 tables are arrayed in a circle, with dealers behind each table facing the outer rounded side and its five to seven player seats. A plastic card shows the rules in effect, including minimum and maximum bets. There may be many large table groups. The tables enclose a "pit" area in which pit personnel, one of whom is the "pit boss," monitor the tables, with telephones and a computer display on hand. The dealer handles the cards, converts players' cash into chips, pays out wins and collects losses.

The game begins with a shuffle of the cards (standard decks) by the dealer and a cut by one of the players. A player bets by putting one or more chips into the circle in front of him. Playing multiple circles is allowed if one or more adjacent circles are available. The dealer gives each player two cards for a "hand," plus two for himself, the second face up on top of a face-down "hole card." The cards are ranked A, 2, 3, ...9, 10, Jack, Queen, King, with the ace having a value of either 1 or 11 (as desired during play) and face cards count as 10, so we call them 10s. The goal is to have a total as close as possible, or equal to, 21. If the total is greater than 21, the player loses ("busting"), even if the dealer busts too. Players with a low total can ask for additional cards ("hits"), one at a time, until they are satisfied ("standing") or they bust. They have the option of doubling their bet and taking just one card (a "double down"). A hand with a pair (10-cards need not be identical) can be split to make two hands, with an additional card added to each. These are then played normally. Some tables permit doubling down after splitting, some don't. If a split card is dealt a card of the same value, a "resplit" is allowed up to as many as four hands, but a pair of aces may be split only once and each gets only one card. If a player's two original cards are an ace and a 10, that is a "natural," popularly called a Blackjack. Instead of getting even money he gets 1-1/2 times the bet unless the dealer has a natural also, in which case he ties. Other than that a dealer's natural means all players lose, even those with a non-Blackjack total of 21. When the dealer's up-card is an ace, players are offered a side bet (called "Insurance") on whether the down card will give the dealer a natural. They can place up to 1/2 their original bet to take that wager, which pays 2 to 1 if successful.

When all players have played their hands in turn, the dealer plays his. He must hit if his total is less than 17 and must stand on 17 or more. The table rules may require the dealer to hit a "soft" 17 like A,6 (which is advantageous to the house). A soft hand is one that can count an ace as either 1 or 11. Players win (even money), tie, or lose their bet depending on whether their total exceeds, ties, or is less than the dealer's total. If the dealer busts, those who have not busted are winners with any total. A thorough discussion of the rules can be seen at [www.blackjackinfo.com](http://www.blackjackinfo.com), "The World's Most Popular Blackjack Website."

Three skills are required for winning at 21: (1) evaluating the favorability of the remaining cards and betting accordingly, (2) playing hands accurately, and (3) doing (1) and (2) without arousing suspicion of doing them well. In Nevada, a casino can legally bar a player from the game if his skill seems excessive. Skill (3) requires "camouflage," looking and acting like a typical stupid gambler who is destined to be a loser. Using whiskey as a "deodorant" earlier is helpful.

## Evaluating Favorability ("Counting")

The MIT team strategy was used against games having four or more decks dealt out of "shoes," not the single or double-deck games in which cards are hand-dealt. While the odds slightly favor the casino at the beginning of dealing, as cards are dealt the odds fluctuate between favorable and unfavorable to the player. Tens and aces are good for the player (e.g., more naturals and successful double downs), while small cards are bad (the dealer is less likely to bust). Recognizing when the remaining cards favor the player is the aim of "counting." The book refers to their use of "a highly technical count" to evaluate favorability. Well, the count they used is the very simple Hi-Lo, with cards 2 through 6 counted as +1, ace and 10s as -1. It is a count suggested for beginners, not as accurate as some others.

A so-called "counter" using Hi-Lo merely maintains a running count of the number of low cards seen, while subtracting the number of aces and 10s. When the total is positive, the remaining cards are favorable to the player. How favorable? The count derived during play is called the "running count," with must be adjusted to make it a "true count." This means dividing the count by the number of decks remaining. A running count of +18 is a true count of +9 if two decks remain undealt. The rule of thumb is that each positive true count represents a player advantage of 1/2 of one percent. Since a shoe is typically that unfavorable to begin with, the true count must be reduced by one before judging the current favorability. A true count of +9 becomes +8, which multiplied by .005 is .04, which means a 4 % advantage for the player.

## Bet Sizing

Obviously the size of a bet should be related to both the favorability of the wager and the available bankroll. If you bet too much when the odds are in your favor, you can endanger your bankroll; bet too little, and you miss out on some good profits. It therefore is logical to practice "fractional betting," wagering fractions of bankroll based on the favorability of the bet. A popular principle for sizing bets in favorable situations is known as "Optimal Betting" (OB), aka the "Kelly Criterion," named after J. L. Kelly Jr, who published it in *The Bell System Technical Journal* in 1956. Its goal is to maximize the probable growth rate of bankroll. This is equivalent to having a logarithmic utility function, as the probability of losing half of the bankroll is the same as for doubling it, an attractive balancing of moderate risk with large potential gain. If that seems too risky, betting a smaller fraction reduces the chance of losing half the bankroll, while increasing the time required to double it. Many find that more comfortable. OB is explained in detail on my web site, under "Blackjack Topics."

If a wager is simple win-loss, the OB bet size is a fraction of current bankroll that equals a bet's mathematical advantage ("edge"). When flipping a coin biased 51-49 (2% edge) in favor of heads, the OB for heads is 2% of the current bankroll for each flip. Those who would bet 4% could make a lot more money, but the most likely outcome is that their bankroll would be unchanged in the long run (after large swings of fortune). Those who would bet an even larger percentage are most likely to end up losers in the

long run, and the swings will be even more extreme.

If the possible results of a bet have a wide variance, as with a complex game like 21 (which can have multiple payoffs or payouts per hand that exceed the original bet size), the advantage must be divided by the variance (standard deviation squared) of the results to determine OB. The variance of 21 depends on the rules in force, including the number of decks, but is typically between 1.2 and 1.3, let's say 1.25. When the advantage is .04, that must be divided by 1.25 to get the right fraction for betting, which is .032 (3.2% of bankroll).

Since each positive Hi-Lo true count (after subtracting one for the house) represents approximately a 0.5% advantage, we can say that OB dictates a bet size of  $.005 / 1.25$  of bankroll  $\times$  (true count - 1). For a \$25,000 bankroll, the OB would then be  $.004 \times 25000 = 100$ , indicating an OB of \$100  $\times$  (true count - 1). If the bankroll grows to \$50,000, the OB would double to \$200  $\times$  (true count - 1), and if the bankroll shrinks to \$12,500 (just as likely) the OB would become \$50  $\times$  (true count - 1). So it boils down to this: Establish a proper "unit" for OB purposes, which is  $.004 \times$  current bankroll. Then multiply the unit value  $\times$  (true count - 1) to get the right bet size. Of course you can't conveniently bet units of \$27 or \$58, so just round down to chip values, \$25 and \$50 in those cases. Rounding down is safer than rounding up, which would be overbetting.

## Playing Strategy

The majority of 21 games have a very small house advantage, less than 1%. This assumes a player uses a set of rules called "basic strategy" that was developed with the help of computers in the early 60s. For each two-card hand and each dealer up-card, basic strategy dictates whether you hit, stand, double down, or split a pair. Very few know basic strategy, even though it takes only an hour or so to learn, plus perhaps a few hours practice. The typical gambler plays so badly that the house makes about 4-5% on his bets. One reason is that many correct plays are not intuitively obvious. For instance, basic strategy says that a pair of 8s should always be split, even when the dealer shows a 9, 10, or Ace; A7 ("soft" 18) should be hit vs dealer's 9, 10, or Ace; and the Insurance bet should not be taken regardless of one's hand. Few players know these things, even though tables of basic strategy are widely available. The web site *blackjackinfo.com* provides an attractive layout for any set of rules, including number of decks used.

Optimal playing strategy often differs from basic strategy, as many actions depend on the current true count. Splitting 10s violates basic strategy, but is correct against dealer's faced 6 when the Hi-Lo true count is +4 or more in a 4-deck game. If he wants to maximize profits, a counter must both play and bet according to the count. That is much harder than just using basic strategy, as many strategy tables incorporating count requirements for various non-basic plays must be memorized. Actually there is not much to be gained from most strategy variations, but those justified only by a high true count should be learned because there could be a lot of money on the table. With an original bet of \$150 and dealer showing a 6, a high count justified my splitting and resplitting 10s until I had four hands. The dealer busted, so I won \$600. Had I played basic strategy, merely standing with the two 10s, I would have won only \$150. Sound good? Consider that losing \$600 was a good possibility too, even though the odds favored my action. The most profitable deviation from basic strategy, by far, is taking Insurance when the Hi-Lo true count is +3 or more. It is then likely that more than 1/3 of the remaining cards are 10s, making the 2-1 payoff for a winning Insurance bet favorable to the player.

## Tax Consequences

The IRS requires that gambling winnings be reported as income, but losses can be deducted up to the amount of winnings. Winnings must be recorded as gross income but losses are claimed as itemized deductions, so reporting net winnings is not allowed. Deducting expenses related to winnings is very problematical, but non-personal expenses like airfare might pass if a player spends something like eight hours a day in the casinos. Court cases in which bettors tried to deduct expenses have been weak and unsuccessful. In any event, reported losses and related expenses may not exceed the amount of winnings.

Good record keeping is essential. The famous bridge player Oswald Jacoby was unable to substantiate losses claimed to offset unreported winnings of approximately \$200,000. His income and losses were reconstructed by the IRS using bank records. For failing to keep good records he was hit with a \$50,000 negligence penalty, later reduced. However, the courts have been inclined to give credible taxpayers some leeway concerning inadequate records. Records can be restricted to sessions of play, even though a literal reading of the law would require each bet's win/loss to be recorded--quite impossible. Realistically, the casual bettor's gambling activities are not likely to come to the attention of the IRS unless he is "lucky" enough to win a large amount.

Large slot and Keno winnings are automatically reported to the IRS and withholding taxes apply. Table games are exempt from this requirement. Instead they are subject to the Currency Transaction Report by Casinos (CTRC) Casinos. A CTRC must be filled out for cash and chip transactions (in or out) with players that total \$10,000 or more in a single day. Subjects of a CTRC must have their identity reliably verified, and the CTRC includes the social security number. Moreover, proofs of identity must be re-verified periodically. Kevin says he used "legal aliases" but his true social security number. All aliases are legal unless used for defrauding, opening a bank or credit account, or sidestepping any legal obligation, so I guess this usage was legal.

## MIT Team Play

A more efficient way to beat the game than playing one-on-one against the dealer is to form a team. Members of the MIT team described in the film and book were of three types: Spotters, Gorillas, and Big Players (BPs). Spotters sit at different tables making minimum bets and counting, signaling a nearby Gorilla or BP when the count is quite high. That count is relayed by a clever oral code to the entering Gorilla or BP so he will know how much to bet. In the film the Spotter crosses hands in back to indicate a high count, which must look rather strange to casino personnel. In the book the Spotter folds his arms, which is more reasonable.

The Gorilla is a high-profile seemingly wild bettor, pretending to be drunk, perhaps loud and obnoxious, with one aim of driving other players from the table so he doesn't have to share a rich shoe with them. Getting the minimum bet size increased does *not* have this effect (as the book claims), since a "grandfather" policy lets existing players continue with lower bets. The Spotter relays the *true* count (I assume) to the Gorilla, who does not count and makes sure the pit boss knows that. Instead he relies on the Spotter, who stays at the table, to count and signal when to raise or lower the bet. It isn't said explicitly, but apparently the Gorilla plays basic strategy.

A BP enters the table similarly, but is signaled the current *running* count (I assume), which he continues to update on his own without help. He too should be a good actor, looking like an obnoxious high roller who throws money around. Unlike the Gorilla, he must be a very strong player who knows how to adjust bet size and vary playing strategy in accordance with the true count. The reason for a Gorilla class

apparently is that skillful BPs are rare. The Spotters usually stay at the table when the BP sits in, evidently in order to verify the BP's wins or losses. However, Spotter play consumes good cards that would otherwise go to the BP.

## Ken Uston's Team

Ken Uston, one of the best 21 players ever, had numerous teams playing in the 1980s, as described in his 1986 book, *Ken Uston on Blackjack*. There were only two classes of player, Counters and BPs. They worked together at a table, the Counter signaling the BP to join the table by putting hand on cheek. Thereafter the Counter continued counting, signaling the BP when to raise or lower his bet and how to play (hit, stand, double-down, or split). A BP could drink as much as he wished for camouflage, as long as he bet and played as directed. They took on single and double-deck games as well as shoes (why not?). The count they used was the Uston Advanced Plus/Minus Count, more accurate than Hi-Lo.

For a while Uston and his players used small hidden computers to evaluate the remaining cards and to direct the play of the hand. Naturally the casinos took this hard, and were able to make computer usage a felony, ending that game.

The Counters were very expert, a result of many hours of practice. Still, Uston says the teams had only a 1% overall advantage over the house. They won in only 60% (80% with computers) of their play sessions. Uston writes that one Counter-BP pair could play full-time for a month and still have a one in five chance of losing. Similarly four such pairs could play full-time for a week and have the same chance of losing. Comparing these numbers to MIT team results makes the latter seem miraculous.

Investors got 50% of the winnings, Counters 40% (half based on time, half on results) and BPs 10%. They played every day of the week when active, not just weekends like the MIT team. The amount of money won was in the hundreds of thousands, no talk of millions (as the MIT team claims). Note the sensible distribution of payments to participants. Counters needed much more expertise than the non-counting BPs.

## Casino Countermeasures

If a player seems too skillful, a Nevada casino can legally bar him from playing. Before that happens, "heat" is usually applied. That consists of pit personnel standing beside the dealer intently watching the player's action. The aim is to intimidate a possible counter so that he leaves voluntarily. Video cameras (the "Eye in the Sky") are ubiquitous, and those monitoring table action can zoom in on any player or dealer. The pit personnel and video surveillant are in constant contact, so that either can tell the other to watch some player closely. Videos are recorded and can be replayed. Identifying counters is just one concern. They are also interested in spotting real cheaters (e.g., players who mark cards so they can tell what is coming out of the shoe before they hit or double down) and cheating dealers (who may make mistakes to benefit a friend or palm a chip for themselves). Just recently two casino dealers in Reno were arrested on suspicion of cheating. They are also looking for innocent dealer mistakes.

Much has been made of face-recognition computer software in use by casinos, which was mentioned in the book, but it is not accurate unless the subject is facing forward and well-lit. A change in pose, lighting, expression, or age; low resolution because of distance; and accessories such as a beard, long hair, or glasses, can fool the system. Maybe in 10 years it will work well, but not now, and certainly not

in the time of the MIT team. The main recognition tools are a book of photos shared by the casinos and the fantastic memory of pit personnel.

Both the film and the book have the MIT team adopting disguises when recognition has become a problem. They had it backwards. When people look at a face and put it in their memories, it is the main feature of the face that gets into their memory bank. That would be eyes usually, perhaps a very large nose, or for men, facial hair. When a clean-shaven man grows or dons facial hair, beard or mustache, or dons a wig, thinking to become unrecognizable, he still gets recognized because his eyes haven't changed. However, if starting out with facial hair, which is then the main recognition factor, shaving it all off later leaves casino personnel with no identifying clue. Dark glasses are not a good idea, as counters are known to use them in order to hide their interest in the cards of others.

The film has Ben being severely beaten (on the casino premises, and with an MIT ID card in his wallet) by members of a detective agency that casinos hired to detect cheaters and counters. This would never have happened in that time period. Successful legal action by those roughed up in previous decades (usually for cheating, not counting) pretty well ended the physical danger.

## Further Comments on the Film and Book

What first caught my attention was the simple Hi-Lo count used by the MIT team. Surely these smart people could employ a more accurate count system. Still, Kevin, learning about counting, has a copy of Edward O. Thorpe's *Beat the Dealer*, a 1966 book that featured the Hi-Lo count and was very outdated by 1994, when the MIT story begins. I would have expected something more current, like *Professional Blackjack*, by Stanford Wong, and *The Theory of Blackjack*, by Peter Griffin, which greatly advanced knowledge of the game, or at least Uston's book.

The hero of the film, Ben Campbell (Kevin Lewis in the book), is first seen in a math (Non-Linear Equations) classroom, solving the "Game Show Host Problem" posed by the professor. Featured in a television show, the problem arises after a contestant guesses that the big prize, a car, is behind one of three doors. The host then opens another door, which shows a goat, and asks the contestant if he wants to switch his choice to the third door or stick with his original choice. It doesn't take a genius to see that it's right to switch, but the film suggests that seeing that is brilliant. Ben says his correct answer was "based on statistics," but statistics had nothing to do with it. The 1/3 chance that his first choice was right has not changed, so it's 2-1 that the car is behind the third door. Simple arithmetic, not statistics.

In the film the team is seated in a restaurant booth and "Jill" is asked by another girl whether it is correct to split 8s when dealer's up-card is a 10. To my surprise the answer was, "Against a five or six, it's okay, but against a 10 it's a sucker bet." And math professor Micky, the team's adviser, says she's right! This is ludicrous, something a complete ignoramus might say, as basic strategy is to split 8s always. If these people need a consultant for some future film, I am available for a cheap price.

The film made no reference to a Gorilla, which would have been an unnecessary complication. Ben starts out as a BP on his first trip (!) with the team, also acting as the "Donkey Boy" who carries hundreds of thousands of dollars in his underwear to avoid having it scanned. With a five-person team, why would this task not be shared, instead of having one person with bulging pants going through security? It is common for gamblers to take large amounts of money on their person to Las Vegas, and it is not illegal

to do so unless the flight is going outside the USA. It is probably true, however, that a very big amount might bring in the dog sniffers, and if the slightest trace of drugs is detected the money will be confiscated. But why are they taking so much to Las Vegas? Most of it comes from winnings there, evidently, so why not keep it in a safe deposit box in Las Vegas? Speaking of money, both the book and the film have Ben/Kevin storing hundreds of thousands of dollars in his MIT dorm. Again, have they no safe deposit boxes in Boston? Of course the money gets stolen, conveniently solving the problem of explaining what happened to all the money.

In the book a player grabs drinks from a tray carried by a passing cocktail waitress, one time two vodka martinis and another time a scotch. Has the author ever tried that? Those trays of drinks are for people who have ordered them. If you grab one, the waitress will yell for Security. If you want a drink, you ask a waitress to bring you one. In the book Kevin is playing with three empty martinis glasses in front of him. No one can play well on that much alcohol. In the film Ben is advised by Micky to order "a tonic water with lime at the table and act a little drunk." A smart counter would never do that. You bring a non-alcoholic drink *to* the table-- you don't order one *at* the table-- if you want to act a little drunk. Later in the film it is the tonic water order, together with the Spotter's obvious signal, that catches them when security replays the video of their table. The minimum bets of the Spotter were also noticed, as very few people play the minimum on every hand. They also remarked that Ben did not double down with 11 "when the count was lighter." Why would a BP stay at the table with a negative count? The film has Ben entering a table when signaled, despite the fact that another team member (a Gorilla?) was playing next to the Spotter. Three teammates at one table is ridiculous.

The MIT team spends money in Las Vegas like there is no tomorrow, with Micky in a top-floor suite that includes a grand piano! Team members also stay in nice suites, often in a casino where they play. They spent "a small fortune" on high-quality disguises. Ben frequently tips \$100, evidently to give the appearance of being a high roller. They must have had at least one losing trip, but there is only one instance shown of losing at the table. Ben loses control, disobeys the Spotter's instructions, and loses over \$200,000. (In the book it was just bad luck, and it was \$100,000.) Other than that, the BP seems to win every bet. The game isn't like that. Even with an advantage you lose more often than you win, coming out ahead only because the wins average more money than the losses. The book says a Gorilla's advantage can be "staggering" when the count is high. Given a rare true count of +11, the advantage is a relatively large 5%, but is that "staggering"?

The book says that investors (who put up the bankroll) earn 12% on their investment--12% per what? Per weekend trip, per year, or what? It also states that card-counting literature says the rate of return for counters is 2%. I can't recall such a statement, which anyway is meaningless if a time period isn't attached. The film says 50% of winnings go to Micky, nothing about investors. In his essay at the end of the book, Kevin says investors never received less than a 30% return per year for five years straight. It's all very confusing.

The film shows the team arriving from the airport in a limo and entering the Riviera casino as a group. Teams should not do that; they should go around alone, with a home base in some cheap out of the way place. Why do things that attract attention?

One can count many shoes, a tedious process, without getting a very high count. The book says 4 to 10 Spotters were used. With only four Spotters, the MIT team would be lucky to get a big plus count more than once an hour. So what does the BP do while waiting for a signal? Does he wander around and around? Then he always jumps into a table with maybe 40%-50% of the shoe remaining. High true counts

just don't happen earlier in the shoe. The BP never starts play at a shuffle, and he always leaves at shuffle time. Casino personnel would have spotted such team play very quickly. They had learned a lot from the discovery of Uston's activity.

The book discusses shuffle tracking, in which a clump of known high or low cards is followed through the shuffle procedure. A player can then cut the decks so that a high-card clump will come into play or a low-card clump will not. When a high-card clump is due to come out while playing, a bigger bet becomes justified even if the running count is not high. I may have been the first to publish an account of this procedure, but publishing had a bad outcome. It seems that casino people read publications such as my favorite, *Blackjack Forum*. Before long the shuffle procedure had been universally changed to ensure a thorough mixing of all the decks, and my counter friends were not happy with me. Since then I have never seen a poor shoe-shuffling technique, and I have to doubt that the MIT team saw any, at least not in Las Vegas.

Another ploy I enjoyed in the 80s was to notice the last card in the shuffled decks, before they were offered for cutting. If the card was high, a deep cut ensures that it comes into play, and if low, a shallow cut ensures that it does not. There was no possibility of cutting so the card went to a specific player, or to the dealer, as the book says is possible. It might have been done if you were allowed to cut very close to the end of the assembled decks. They didn't let you do that, as the minimum cut was 15 cards. In the book a team member cut exactly 52 cards from the back when an ace was the back card. "If you're good, really good," it is explained, "you can get the dealer to deal you that specific card." Huh? Even if you know where the card is, it will go to the dealer half the time if you're alone at the table. If you're not alone, the chance of getting the card is much less. Besides, cutting exactly 52 cards is problematical, if not impossible. If you are one card off and make a big bet, expecting to get an ace, the dealer gets it and of course has a natural. Not long after I discussed end-card cutting in print, the end card was carefully kept out of view at all the major Las Vegas casinos. I don't understand how these people got to see it.

Kevin learns "when to leave the table and when to start burning the cards." The two actions are mutually contradictory. One burns cards by playing multiple hands and making bad hitting and splitting plays, in order to use up cards when the count is very bad. That gets rid of those little cards, with small bets to minimize the consequent losses. But if the count is that negative a BP leaves the table, he doesn't play.

The book states repeatedly that casino games "are rigged," giving the house a "hefty advantage." With the house advantage of decent 21 games in the neighborhood of a minuscule 0.5%, "rigged" is hardly a descriptive adjective. Even craps isn't so bad, with a 1.4% house advantage if one doesn't take any of the sucker bets (e.g. on boxcars, two sixes). There are some bad 21 games, for example tables that pay only 6 to 5 for a natural instead of 3 to 2. Ignorant gamblers crowd those tables, thinking that 6 to 5 is better than 3 to 2! A counter wouldn't sit there, as the rule favors the house by more than 1%.

As a Spotter Kevin was playing against a six-deck shoe from which five decks were dealt. Such depth provides great opportunities for counters when the fifth deck comes out. However, in the MIT time period (late 90s) I doubt there was any Las Vegas casino dealing that deep from six decks. In Shreveport, LA, yes, but not in Las Vegas. When the count goes high, Kevin signals the BP to come in with the count at +15 and two decks remaining in the shoe. The BP "pushed a seemingly random handful of chips into his betting circle: three blacks, two purples, and six green, a total of \$1,450. *Let the Eyes in the Sky figure that one out!*" Never happen. Dealers are required to restack chips in order of size before dealing, largest on the bottom, for the benefit of the pit boss. Kevin is dealt a 10 and a 9, the BP a pair of 10s, and the dealer's up card is a 6. The count becomes  $15 - 1 - 2 + 1 = 13$ . Divide by two decks remaining and it's

6-1/2, so the BP correctly splits the 10s, receiving a 10 on one and a 9 on the other, and the dealer busts. However, he should have split the first hand again with its two 10s, as the true count was then 5-1/2 (actually more, with fewer cards left in the shoe) and Hi-Lo strategy is to split with +4 or more in this situation. Later he hits with 16 facing dealer's 2, which means the count must have been -9. Then he "takes advantage of a hot streak" and raises his bets. A hot streak means lots of 10s and aces, lowering the count and calling for a bet reduction, not an increase. That was an awful lot of count change within a single deck (the sixth deck not dealt). It doesn't seem possible.

Kevin Lewis (real name Jeff Ma) adds an essay to the back of the book. In it he says that a betting unit should be 1% of the available bankroll. As discussed above, it should be 0.4% unless you want to take the chance of overbetting, not a wise policy. He seems to be unaware of the OB principle and the variance effect. He also says you should play two hands when the count is favorable. That is good advice when not alone at the table, as you want to get more of the good hands than others do. Playing alone with the dealer it's not the best policy. If you play two hands, they are not independent because there is a covariance of about 0.5. You have to reduce each bet by 25% to have the same amount of risk. If the count says to bet \$600, OB says to bet \$450 on each of two hands, getting 50% more money on the table. However, you are also getting more cards dealt per round, which will mean fewer rounds. The effect comes out about even, except for one thing: the count gets updated more often when playing a single hand, and that is advantageous. When there is only one more round expected before the shuffle, then it makes sense to play multiple hands. Proper bet sizing for three or more hands is provided by Stanford Wong in *Professional Blackjack*.

With all the mistakes, inconsistencies, and obvious exaggerations in the MIT story, it is hard to take it seriously. There was such a team, and it probably made money, but not nearly as much as claimed. The book says they made almost half a million dollars in one weekend. Ken Uston must have laughed if he read that. It is a fun piece of near-fiction, however. I recommend both the film and the book.